High Resolution Tactile Imaging Sensor using Total Internal Reflection and Non-rigid Pattern Matching Algorithm

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Abstract—A novel tactile imaging sensor, that is capable of measuring the elasticity of the touched object, is designed, implemented, and tested. In the proposed sensor, a multi-layer Polydimethylsiloxane optical waveguide has been fabricated as the sensing probe. The light is illuminated at the critical angle to totally reflect within the flexible and transparent waveguide. When a waveguide is compressed by an object, the contact area of the waveguide deforms and causes the light to scatter. The scattered light is captured by a high resolution camera. To find the elastic modulus of a touched object, multiple tactile images are taken from slightly different loading force values. The applied force has been estimated using the integrated pixel values of the tactile image. The strain has been estimated by matching the series of tactile images using the proposed non-rigid pattern matching algorithm. The measurement method was validated by the commercial soft polymer samples with the known elastic modulus. The experimental results showed that the tactile imaging sensor can measure the elastic modulus with the error less than 5.38%.

Index Terms—Tactile Sensor, Artificial Tactile Sensing, Pattern Matching, Image Registration, Elastic Modulus Measurement.

I. INTRODUCTION

TRADITIONALLY, physicians have used palpation to detect breast or prostate tumors. Unhealthy tissues tend to be stiffer (low elasticity) than the healthy tissues [1], [2]. To help physicians detect tumors more efficiently, various imaging techniques utilizing different modalities such as computer tomography (CT), ultrasonic imaging (US), magnetic resonance imaging (MRI), and mammography (MG) have been developed [3], [4], [5], [6]. However, each of these techniques has disadvantages: harmful radiation to the body (CT, MG), low specificity (MRI), complicated system (US, MRI), etc. Moreover, these techniques can only provide the spatial information of the tumor in the tissue. They do not measure the mechanical property (e.g. elastic modulus) directly [7]. The absolute mechanical property is very important in detecting the severity of the tumor. Using the elastic modulus, we can even differentiate the benign and malignant tissues [8]. Thus we design, implement, and test a tactile imaging sensor that quantifies the elastic modulus of the contacted object.

To measure the elastic modulus of tissues, different modalities have been investigated. The sonoelastography uses ultrasonic longitudinal compression waves to differentiate and characterize breast tissues [7], [9]. However, the resolution and contrast of images produced by the sonoelastography is very low and contains much noise. In addition, a highly trained technician is needed. Some research groups use force sensing resistors and a super-resolution algorithm for a palpation device [10]. However, this approach can only detect the relative stiffness, not the absolute elastic modulus of the tissue. In addition, even though spatial resolution of the tactile sensing has been improved by the super-resolution algorithm, it has still fairly low resolution compared to human fingertips, which have millions of mechanoreceptors per square inch of the skin [11]. Some tactile sensors use piezoelectric cantilevers for the elastic modulus measurement [12], [13]. However, this method requires sensitive calibration [14], [15]. In addition, this method provides low spatial resolution image due to the size of the sensing probe. Our tactile imaging sensor measures the elastic modulus of the contacted object with the high spatial resolution. The device is also simple and easy to use.

In this paper, we present a novel and simple palpation device, tactile imaging sensor. In the current design, a Polydimethylsiloxane (PDMS) is used to make a multi-layer flexible transparent waveguide. The waveguide is the main sensing probe. In order to maximize the tactile spatial resolution, we utilize the total internal reflection principle. For this purpose, micro light emitting diode (LED) light sources are attached along its edges of the waveguide. Then the light is illuminated at the critical angle to totally reflect within the waveguide. Once the waveguide is deformed, it causes the trapped light to change the critical angle and diffuse outside the waveguide. The scattered light is captured by a high resolution camera. The salient feature of this sensor is its capability of measuring the elastic modulus of the touched object without any external force sensor. The elastic modulus is defined as the slope of its stress and strain curve in the elastic deformation area [16]. To find the elastic modulus, the applied force has been estimated through the integration of pixel values of the tactile image [17]. For the strain estimation, series of tactile image are captured under different force magnitudes. Then the displacements between random patterns recorded in tactile images are estimated using the proposed non-rigid pattern matching algorithm. The obtained stress and strain information are finally used to identify the elastic modulus of the compressed object.

In the following section, the tactile imaging sensor design,
sensing principle, and sensor specification are introduced. Then, the proposed elastic modulus measurement method based on the tactile images is discussed. Also a new non-rigid pattern matching algorithm for the strain estimation is discussed. Next, the proposed measurement method has been validated using the commercial rubber samples with different elasticity. Finally, the conclusions are presented.

II. TACTILE IMAGING SENSOR DESIGN AND SENSING PRINCIPLE

In this section, we present the design concept and sensing principle of the proposed sensor in detail.

A. Sensor Design

The tactile imaging sensor incorporates a planar optical waveguide unit, a light source unit, a light coupling unit, a high resolution camera unit, and a computer unit for data acquisition and analysis. Fig. 1 shows the schematic of the proposed sensor.

![Schematic of the tactile imaging sensor](image)

The optical waveguide is the main sensing probe. The waveguide is composed of Polydimethylsiloxane (PDMS, Si(CH$_3$)$_2$), which is a high performance silicone elastomer [18], [19]. In the current design, the waveguide needs to be flexible and transparent and PDMS meets this requirement. The human skin is composed of three layers with different elastic modulus. They are epidermis, dermis, and subcutaneous. To emulate the human touch, three different densities of PDMS are stacked together. The elastic modulus of each PDMS layer is matched as the modulus values of epidermis of PDMS are stacked together. The elastic modulus of each layer is 2 mm for PDMS layer 1, 3 mm for PDMS layer 2 and 5 mm for PDMS layer 3, respectively. The fabricated optical waveguide is shown in Fig. 2(a).

![Fabricated optical waveguide](image)

The digital camera is a mono-cooled complementary CMOS camera with 8.4 μm × 9.8 μm individual pixel size (Guppy, Allied Vision Technology, Germany). It has a pixel array of 768 (H) × 492 (V) with 8 bit bit-depth and its maximum resolution is 0.4 megapixel. The camera is placed below the optical waveguide. A heat-resistant borosilicate glass plate is placed between the camera and the waveguide to sustain the waveguide without losing the camera resolution.

The internal light source is a micro-LED (Unique-Leds, Newalla, OK) with a diameter of 1.8 mm. There are four LED light sources placed on four sides of the waveguide to provide enough illumination. Light is directed into the waveguide using an optical coupler which allows light to enter the waveguide as much as possible. A number of methods are available for that purpose, such as direct coupling, prism coupling, grating coupling, and tapered coupling. The simplest is direct focusing with a lens, in which light is focused onto the waveguide edge with an optical lens. Coupling efficiency is determined by how well the input light’s profile matches that of the mode. If the profiles are matched and there is no discontinuity, light from the source will just continue propagating into the waveguide. However, if there is a discontinuity, only some portion of light would be coupled, while the rest would be diffracted away. Light coupling minimize this diffraction light. In the current design, we use plano-convex lens (Newport Corp., Irvine, CA) as the optical coupler. The lens is 12.7 mm in diameter and a focal distance 12.7 mm. The direction and incident angle of light have been calibrated to be trapped inside the waveguide. The sensing principle and optimal light incident angle are discussed in the next section. Fig. 2(b) shows all units integrated tactile imaging sensor.

B. Sensing Principle

The proposed sensor operates on the principle of total internal reflection (TIR). According to the Snell’s law, if two mediums have different refraction indices, and the light is shone throughout those two mediums, then a fraction of light is transmitted and the rest is reflected [21]. The angle above which the light is completely reflected is the critical angle. Since the waveguide is surrounded by air, having a lower refractive index than any of layers in the waveguide, the incident light directed into the waveguide can be trapped inside the waveguide. The basic principle of the sensor lies in the monitoring of the reflected light caused by the changing of the critical angle by the contacted object. Fig. 3 illustrates the conceptual diagram of the sensing principle.

To trap the light inside the waveguide, the necessary critical angle and acceptance angle of light are analyzed using the geometric optics approximation. In this approximation, we assume the light wave as a ray. This allows determining...
the direction of light illumination. Consider the geometry as shown in Fig. 4. The waveguide is made of three PDMS layers on top of glass plate.
1) Layer 1: PDMS, refractive index $n_1$.
2) Layer 2: PDMS, refractive index $n_2$.
3) Layer 3: PDMS, refractive index $n_3$.
4) Layer 4: Glass plate, refractive index $n_4$.

$n_0$ and $n_5$ are the refractive indices of air and are equal to 1. The layers are positioned in the order of decreasing refractive indices, so $n_1 > n_2 > n_3 > n_4 > n_0 = n_5$.

The TIR condition has been achieved when

$$\theta = 90^\circ - \gamma$$

where $\gamma$ is the highest angle, under which the light directed into the waveguide remains trapped inside it. The acceptance angle $\theta_i$ is the maximum angle, under which the light scattered as the waveguide deforms according to the applied force. The propagation angle $\gamma_i$ are related to the acceptance angle $\theta_i$ by the same Snell’s law:

$$\sin \theta_i = n_i \sin (90^\circ - \gamma_i) = n_i \cos \gamma_i.$$  \hspace{1cm} (6)

Further, transforming Eq. (6), we obtain

$$\sin \theta_i = n_i \cos \gamma_i = n_i (1 - \sin^2 \gamma_i)^{1/2} = (n_i^2 - n_i^2 \sin^2 \gamma_i)^{1/2}. $$ \hspace{1cm} (7)

But from Eqs. (1) to (5), all $n_i \sin \gamma_i$ are equal to $n_0$, which is equal to 1 for air. Therefore, we finally have the acceptance angle $\theta_i$ for each layer $i$:

$$\theta_i = \sin^{-1}[(n_i^2 - 1)^{1/2}].$$ \hspace{1cm} (8)

Light, incident on layer $i$ under the acceptance angle $\theta_i$, will be trapped inside the waveguide.

In the current design, the refractive index of each PDMS layer and glass plate are measured approximately as 1.41, 1.40, 1.39, 1.38 and the acceptance angles $\theta_i$ are calculated as $\theta_1 = 83.73^\circ$, $\theta_2 = 78.46^\circ$, $\theta_3 = 74.89^\circ$, and $\theta_4 = 71.98^\circ$. Thus for the TIR in the waveguide, the spatial radiation pattern of LED light with the angle less than $71.98^\circ \times 2 = 143.96^\circ$ has been chosen and placed to inject the light.

![Conceptual diagram of the sensing principle. The light scatters as the waveguide deforms according to the applied force.](image)

![Graphic representation of light propagation as a ray, propagating in the waveguide at propagation angles $\gamma_i$, $i = 0, 1, 2, 3, 4, 5$.](image)

Due to Snell’s law, the propagation angles $\gamma_i$ in each layer $i$, $i = 0, 1, 2, 3, 4, 5$ are bound by the following relations:

$$n_1 \sin \gamma_1 = n_0 \sin \gamma_0,$$ \hspace{1cm} (1)

$$n_2 \sin \gamma_2 = n_1 \sin \gamma_1,$$ \hspace{1cm} (2)

$$n_3 \sin \gamma_3 = n_2 \sin \gamma_2,$$ \hspace{1cm} (3)

$$n_4 \sin \gamma_4 = n_3 \sin \gamma_3,$$ \hspace{1cm} (4)

$$n_5 \sin \gamma_5 = n_4 \sin \gamma_4.$$ \hspace{1cm} (5)

The TIR condition has been achieved when $\gamma_0 = \gamma_5 = 90^\circ$ at the boundaries between waveguide and air. Light propagating in the waveguide with angles $\gamma_i$ or higher in their respective layers will be trapped inside the waveguide. The critical angle indicates the minimum propagation angle $\gamma_i$. To make the propagation angle $\gamma_i$ above the critical angle, the acceptance angle $\theta_i$ for the incident light to the waveguide has been calculated.

![The sample tactile image of spherical object made with rubber with diameter of 2 mm is obtained under 0.7 N normal force. Fig. 5(a) shows the initial grey scale tactile image. In Fig. 5(b), a color scale replaced the original gray into color scale for the better visualization. Finally, in Fig. 5(c), the 3-D reconstruction is performed using the “shape from shading” method.](image)

**C. Spherical Object Tactile Image**

The sample tactile image of spherical object made with rubber with diameter of 2 mm is obtained under 0.7 N normal force. Fig. 5(a) shows the initial grey scale tactile image. In Fig. 5(b), a color scale replaced the original gray into color scale for the better visualization. Finally, in Fig. 5(c), the 3-D reconstruction is performed using the “shape from shading” method. From the sample tactile image, we notice that the light scatters on the contact area and the pixel values of the tactile image distribute in a bell shape, where the pixel intensity is the highest at the centroid and decreases with increasing distance from the centroid.
D. Sensor Characterization

The specification of our sensor is compared with the specificaiton of the human fingertip.

1) spatial resolution between sensing points: The spatial resolution of the tactile imaging sensor is the pixel size of the camera. The spatial resolution between sensing points of the fingertip is at least 0.1 mm, which translates into an approximately 200 × 300 elements grid on a fingertip size area (20 mm × 30 mm) [11]. In the proposed sensor, the pattern discrimination ability is 9.8 μm and translates into an approximately 2041 × 3061 elements grid on the same fingertip size area. This makes the sensor high resolution.

2) Temporal resolution: With regard to the human fingertip temporal resolution, the fingertip vibration bandwidth is a few Hz for separate touches and hundred Hz for sensing vibration. The camera that we chose had a 768 × 492 resolution at 30 frames per second (30 Hz).

3) Force sensitivity: Sensitivity is described in terms of the smallest input physical signal (input) to generate the output electrical signal [18]. The force sensitivity of the proposed sensor is approximately 2.5 × 10⁻³ N compared to the fingertip force sensitivity of 2.0 × 10⁻² N [23].

4) Hysteresis: The human skin is very hysteric. The skin relaxes with time, with an observed length of up to 8 seconds [24]. The response of the proposed sensor is non-hysteric. In addition, the sensor is stable, repeatable and continuous in its variable output signal. Table I summarizes the sensory specification of the human fingertip and tactile imaging sensor.

<table>
<thead>
<tr>
<th>Design Criteria</th>
<th>Human Fingertip</th>
<th>Tactile Imaging Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial Resolution</td>
<td>0.1 mm</td>
<td>9.8 μm</td>
</tr>
<tr>
<td>Temporal Resolution</td>
<td>0-100 Hz</td>
<td>0-30 Hz</td>
</tr>
<tr>
<td>Force Sensitivity</td>
<td>2.5 × 10⁻² N</td>
<td>2.0 × 10⁻³ N</td>
</tr>
<tr>
<td>Hysteresis</td>
<td>High</td>
<td>Low</td>
</tr>
</tbody>
</table>

III. ELASTIC MODULUS MEASUREMENT USING TACTILE IMAGE

The word “stiffness” is used to describe the mechanical property of the touched object obtained by palpation. The stiffness is expressed by the elastic modulus $E$ having units of force per unit area. The stress is defined as the applied force per unit area. The strain is the fraction change in length in response to the stress. The elastic modulus is expressed as stress over strain as below.

$$E = \frac{\text{stress}}{\text{strain}}.$$  \hspace{1cm} (9)

In this section, the elastic modulus measurement method using the tactile image is discussed.

A. Stress Estimation

The stress is measured as force per unit area. In the current design, if the waveguide of a sensor is compressed by an object, it is deformed in the compressed direction. Then the light scatters at the contact area. This light is captured by a camera. Let $I(x, y)$ be the individual pixel value of the captured tactile image. Since $I(x, y)$ is proportional to the stress, $P(x, y)$, it can be expressed as follows:

$$P(x, y) = f(I(x, y)),$$  \hspace{1cm} (10)

where $f$ is the conversion function. If $C$ is designated as the contact area, then the force $F$ is obtained by integrating the stress over the contact area as follows:

$$F = \int_C P(x, y)\,dC.$$  \hspace{1cm} (11)

The relationship between force and integrated pixel value of the tactile image is established via experiments using a loading machine. As shown in Fig. 6, the loading machine that we used have a force gauge (Mecmesin, West Sussex, UK) to detect the applied force to the waveguide. The force gauge has a probe to measure the force from the range of 0 to 50 N with the resolution of 1.0 × 10⁻³ N. Since the camera is an 8-bit digital camera, each pixel has a minimum value of 0 and a maximum value of 255. We attached a small tip with 2 mm radius to the force gauge to compress the waveguide. In this experiment, starting from the initial force of 0 N, the force was increased in the steps of 0.1 N. When the force reached the maximum value of 2.0 N, it was decreased in a stepwise fashion until it returned to 0 N. The resulting diffused light caused by compression of the waveguide was captured by a camera, and the corresponding applied force was measured by the force gauge.

Fig. 7 shows the pixel values along the contact area’s horizontal direction of the tactile image. As expected, the graph is Gaussian shape and the maximum value is at the centroid of the tactile image. The plot of integrated pixel values of tactile image versus applied normal force is shown in Fig. 8. As shown in the result, the sensor exhibits a linear response, good repeatability and low hysteresis. The curve shows the monotone increasing relationship between the force and the integrated pixel value of the tactile image. The hysteresis loop is not observed in the graph. Using this approximation, we can approximate the applied normal force
from the tactile image. Since the stress is measured as force per unit area, the final estimated stress, \( \hat{P} \), is as follows:
\[
\hat{P} = \frac{F}{C},
\]
(12)
where \( F \) is the contact force obtained from the fitted line in Fig. 8 and \( C \) is the contact area.

Fig. 7: The pixel value along the contact area (horizontal direction) as the normal force varies.

Fig. 8: The relationship between the normal force and the integrated pixel values.

In this paper, we assume that the measured object is smaller than the probe area. Then the contact area, \( C \), becomes the size of the object. In this case, the contact area, \( C \), is estimated by the light scattering area in the tactile image. The scale factor between the actual size and the image pixel distance is \( 6.79 \times 10^{-3} \) mm per pixel. We obtained this ratio by the calibration.

B. Strain Estimation Using Non-rigid Pattern Matching Algorithm

The other value needed for the elastic modulus is the strain. Strain is the geometrical deformation measure indicating the relative displacement between points on the object. Thus if we know the displacement of any particular set of points on tactile images obtained under different loading forces, then we can find the strain of the compressed object. To find the displacement, an efficient non-rigid pattern matching algorithm has been designed. The essence of this algorithm is to automatically measure the displacement by tracking the change in position of control points extracted from two different tactile images [25], [26]. Fig. 9 represents the concept of tracking control points between 3-D reconstructed tactile images obtained under two different loading forces on the same object.

Fig. 9: 3-D reconstructed tactile images of Versaflex CL2000X soft polymer obtained under 0.7 N and 1.2 N normal forces. The strain can be estimated by tracking control points extracted from surface of two different tactile images.

Now the non-rigid pattern matching algorithm is described. First, the point correspondence is found. Second, the transformation function is calculated based on the obtained point correspondence. Finally, the Lagrangian strain is estimated using the transformation function. Let two 3-D reconstructed tactile images obtained under different loading forces as \( O_1 \) and \( O_2 \). From the surface of \( O_1 \) and \( O_2 \), a number of control points are extracted. Let \( P = \{p_1, p_2, ..., p_I\} \), \( p_i \in \mathbb{R}^3 \) be a point set extracted from \( O_1 \) and \( Q = \{q_1, q_2, ..., q_J\}, q_j \in \mathbb{R}^3 \) be a point set extracted from \( O_2 \). If the object is deformed by the contact of the sensing probe, the distance between the points changes, especially when points are far apart. However, the local adjacent points of each point will not change much due to physical constraints [27]. So we define the local adjacent points of each point. For a given point, \( p_i \in P \), one can select adjacent points \( N^a_p \), \( a = 1, 2, ..., A \), which are in the circle centered at \( p_i \). We set the radius of a circle as the median value of all Euclidean distances between point pairs in \( P \). Similarly, for a point, \( q_j \in Q \), adjacent points are \( N^q_q \), \( b = 1, 2, ..., B \). We determine the fuzzy correspondence matrix \( M \). Each element of \( M_{p,a} \) has continuous value between [0,1] and it indicates the correspondence weight between \( p_i \) and \( q_j \). Same as \( M_{p,a} \), \( M_{N^a_p,N^q_q} \) has continuous value between [0,1] and it indicates the correspondence weight between \( N^a_p \) and \( N^q_q \).

The optimal match \( \hat{M} \) is found by maximizing the energy function as follows.
\[
\hat{M} = \arg\max_M E(M),
\]
(13)
where
\[
E(M) = \sum_{i=1}^{I} \sum_{b=1}^{B} \sum_{j=1}^{J} \sum_{a=1}^{A} M_{p_i,q_j} M_{N^a_p,N^q_q}
\]
(14)
subject to \( \sum_{i=1}^{I} M_{p_i,q_j} = 1 \), \( \forall i \), and \( \sum_{j=1}^{J} M_{p_i,q_j} = 1 \), \( \forall j \), and...
$M_{p,q_i} \in [0,1].$

1) **Point Correspondence:** Initially, each point $p_i \in P$ is assigned with a set of matching probabilities using the shape context distance [28]. After the initial probability assignment, the relaxation labeling process updates the matching probability [29]. The relaxation labeling process will assign a matching probability that maximizes $E(M)$ under the relaxed condition as $M_{p,q_i} \in [0,1].$ At the end of the process, it is expected that each point $p_i \in P$ will have one unambiguous matching probability.

For the relaxation labeling process, we define a new compatibility coefficient. The proposed compatibility coefficient quantifies the degree of agreement that $p_i$ matches to $q_j$ and $N^p_i$ matches to $N^q_j.$ It is measured by the vector set from each point to all other points. In the non-rigid degradation of point sets, we note that a point set usually changes their location, but the neighborhood structure of each point is preserved due to the physical constraint. The displacement of a point and its adjacent point constrain one another. Thus, if the distance and angle of a point pair $(p_i,N^p_i)$ and its corresponding point pair $(q_j,N^q_j)$ are similar, we say that they have high correlation. This assumption becomes more accurate if a point pair $(p_i,N^p_i)$ is closer to each other. To quantify this knowledge, we introduce the similarity constraint $\alpha, \beta$ as well as the spatial smoothness constraint $\gamma.$

The first constraint, $\alpha$ and $\beta,$ is the similarity which is related to the distance and angle of $(p_i,N^p_i)$ and $(q_j,N^q_j).$ This first constraint indicates that if $(p_i,N^p_i)$ has smaller distance and angle differences with $(q_j,N^q_j),$ then they are more compatible. The disparities between $(p_i,N^p_i)$ and $(q_j,N^q_j)$ are defined as follows [30].

$$\alpha(p_i,N^p_i; q_j,N^q_j) = \left(1 - \frac{d_i(p_i,N^p_i) - d_j(q_j,N^q_j)}{\max_{i,j} d_i(p_i,N^p_i), d_j(q_j,N^q_j)} \right),$$

$$\beta(p_i,N^p_i; q_j,N^q_j) = \left(1 - \frac{l_i(p_i,N^p_i) - l_j(q_j,N^q_j)}{\max_{i,j} l_i(p_i,N^p_i), l_j(q_j,N^q_j)} \right),$$

where $d_i(\cdot)$ and $l_i(\cdot)$ are the Euclidean distance and angle, respectively.

The second constraint, spatial smoothness $\gamma,$ is measured by the distance between $p_i$ and $N^p_i.$

$$\gamma(p_i,N^p_i) = \left(1 - \frac{d_i(p_i,N^p_i)}{\max_{i} d_i(p_i,N^p_i)} \right),$$

where $\max_{i} d_i(p_i,N^p_i)$ is the longest edge of point-adjacent point pairs. Two points $p_i$ and $N^p_i$ are the most salient if $\gamma(p_i,N^p_i)$ is 1 and the least salient if $\gamma(p_i,N^p_i)$ is 0.

We define the total compatibility coefficient by

$$r_{p,q}(N^p_i,N^q_j) = \alpha(p_i,N^p_i; q_j,N^q_j) \cdot \beta(p_i,N^p_i; q_j,N^q_j) \cdot \gamma(p_i,N^p_i).$$

Clearly, $r_{p,q}(N^p_i,N^q_j)$ ranges from 0 to 1. A high value of $r_{p,q}(N^p_i,N^q_j)$ corresponds to high matching probability between $(p_i,N^p_i)$ and $(q_j,N^q_j),$ and a low value corresponds to incompatibility. The support function $q_{p,q}$ in the $k$-th iteration is then given by

$$q_{p,q}^k = \sum_{i=1}^{J} \sum_{j=1}^{I} r_{p,q}(N^p_i,N^q_j) M_{p,q_i}^k N_{p_i}^q N_{q_j}^i ,$$

Note that $r_{p,q}(N^p_i,N^q_j)$ is weighted by $M_{p,q_i}^k N_{p_i}^q N_{q_j}^i$ because it depends on the likelihood of adjacent point pairs matching probability. Finally, the fuzzy correspondence matrix $M$ is updated according to

$$M_{p,q_i}^{k+1} = M_{p,q_i} q_{p,q_i}^k \sum_{j=1}^{J} M_{p,q_j} q_{p,q_j}^k.$$

The relaxation labeling process updates the matrix $M$ continuously until the stopping criterion is met. The final point correspondence matrix $M$ is used to find the transformation function in the next section.

2) **Transformation Function:** Given a correspondence matrix $M$ between $P$ and $Q,$ we can estimate a transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ that is used to map points from $P$ to $Q.$ In this paper, we use the thin-plate spline (TPS) model for representing flexible transformation. [31], [32], because the TPS model is highly effective for modeling the changes of the non-rigid forms [33].

Let $v_i = (x'_i, y'_i, z'_i)$ denote the corresponding location of $p_i = (x_i, y_i, z_i), i = 1, 2, \ldots, n.$ In 3-D interpolation problem, the TPS interpolant $f(x, y, z)$ minimizes the following bending energy

$$I_f = \int \int \left[ \frac{(\partial^2 f}{\partial x^2} + \frac{(\partial^2 f}{\partial y^2} + \frac{(\partial^2 f}{\partial z^2} \right]^2 + 2\left\{ \frac{(\partial^2 f}{\partial x \partial y} + \frac{(\partial^2 f}{\partial x \partial z} + \frac{(\partial^2 f}{\partial y \partial z} \right\}^2 \right] dx dy dz,$$

and the interpolant form is

$$f(x, y, z) = a_1 + a_2 x + a_3 y + a_4 z + \sum_{i=1}^{n} w_i U(||(x_i, y_i, z_i) - (x, y, z)||),$$

where $a_1, a_2, a_3, a_4,$ and $a_5$ are the coefficients and $w_i$’s are the weighting factors. The kernel function $U(r)$ is defined by $U(r) = r^3 \log r^3.$ The boundary conditions are $\sum_{i=1}^{n} w_i = 0$ and $\sum_{i=1}^{n} w_i x_i = \sum_{i=1}^{n} w_i y_i = \sum_{i=1}^{n} w_i z_i = 0.$ A special characteristic of the TPS is that the resulting transformation can be decomposed into a global affine transformation component and a local non-affine warping component. In Eq. (22), the first four terms of the left hand side describe a global affine transformation and the remaining terms describe a local non-affine transformation. Then the linear system for the TPS coefficients can be expressed as follows.

$$\begin{pmatrix} K \ P \\ P^T 0 \end{pmatrix} \begin{pmatrix} W \\ V \end{pmatrix} = \begin{pmatrix} 0 \end{pmatrix},$$

where $K$ is a matrix containing the known coefficients.
where \( K = \begin{bmatrix} 0 & U(r_{12}) & \cdots & U(r_{1n}) \\ U(r_{21}) & 0 & \cdots & U(r_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ U(r_{n1}) & U(r_{n2}) & \cdots & 0 \end{bmatrix} \) and

\[
P = \begin{bmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & y_n & z_1 \end{bmatrix}
\]

Here, \( r_{ij} = ||p_i - p_j|| \) is the Euclidean distance between points \( p_i \) and \( p_j \). \( W \) and \( Y \) are column vectors of weighting factors and coefficients of Eq. (23) and it is formed as \( W = (w_1, w_2, \ldots, w_n)^T \) and \( Y = (a_1, a_x, a_y, a_z)^T \), respectively. \( V = (v_1, v_2, \ldots, v_n)^T \) is any \( n \)-vector. The matrix \( \begin{bmatrix} KP \\ p^T 0 \end{bmatrix} \) is nonsingular and it is invertable. Thus we can determine the coefficients \( W \) and \( Y \) by multiplication of matrices \( \begin{bmatrix} KP \\ p^T 0 \end{bmatrix}^{-1} \) and \( \begin{bmatrix} V \\ 0 \end{bmatrix} \). The weighting factors and coefficients \( W \) and \( Y \) determine TPS interpolants \( f_x(x, y, z), f_y(x, y, z), f_z(x, y, z) \), the \( x, y, z \) coordinates of the transformation. The resulting function \( f(x, y, z) = [f_x(x, y, z), f_y(x, y, z), f_z(x, y, z)] \) maps each point \((x, y, z)\) to its correspondence point \((x', y', z')\). It provides a continuous displacement field between \( O_1 \) and \( O_2 \). Finally, the nonlinear Lagrangian strain tensor component of the uniaxial loading configuration \( e_{zz} \) is determined as follows:

\[
e_{zz} = \frac{\partial f_x(x, y, z)}{\partial z} + \frac{1}{2} \left[ \left( \frac{\partial f_x(x, y, z)}{\partial z} \right)^3 + \left( \frac{\partial f_y(x, y, z)}{\partial z} \right)^3 + \left( \frac{\partial f_z(x, y, z)}{\partial z} \right)^3 \right].
\]

\[ (24) \]

C. Elastic Modulus Measurement from Stress and Strain

To determine the elastic modulus of the contacted object, the strain component \( e_{zz} \) are averaged to yield the average strain \( \bar{e}_{zz} \). Given the applied normal stress \( \bar{P}_{zz} \), the elastic modulus \( E \) of the object is then determined from the following equation,

\[
E = \frac{\bar{P}_{zz}}{\bar{e}_{zz}}.
\]

The obtained tactile images were then reconstructed to 3-D, and control points were extracted from the surface of 3-D reconstructed tactile images. The correspondence and transformation function between control points were estimated using the proposed non-rigid pattern matching algorithm described in Section III.

The stress in the elastic modulus measurement is the force per unit area. The applied force has been estimated using the calibration curve in Fig. 8. By dividing the applied force with the contact area, \( C \), we determine the stress. The calibration curve was obtained using a small, stiff tip indenter. Strictly speaking, the calibration curve is only valid for a stiff material. But we use this curve for the soft polymer, because both materials are homogeneous and isotropic. There will be some errors due to the material differences. The error due to this assumption can be included to the total error. We also note that because the object is smaller than the sensing probe, the contact area, \( C \), is the contact area of the soft polymer. Thus the applied force has been divided by the soft polymer contact area to find the final stress value.

Fig. 10: The relationship between the normal force and the integrated pixel values in response to the different loading machine tip radius.

In this experiment, we have obtained the calibration curve using the loading machine with 2 mm radius tip and used this calibration curve for the 3 mm radius soft polymer measurement. Theoretically, the calibration curve can be used to estimate forces applied on the objects of different sizes. To investigate the sensor’s measurement capability in more samples of bigger size, we find the calibration curve in response to the different tip size of the loading machine. The size of tips we used were radius of 10 mm and 14 mm. We obtained 15 images of each tip size case under different loading forces and calculated the integrated pixel value of each image. Then the linear regression line was obtained. The experimental results are shown in Fig. 10. According to the Hertzian contact theory, the calibration curve should match each other regardless of the different indenter sizes [34]. However, the result shows that there exist some differences between linear regression lines. At 800 mN normal force, the error between two cases is 2.45%. There can be many reasons for this error. The impurity of the PDMS optical waveguide
is one reason. The unequal distribution of lights in the PDMS optical waveguide due to the misalignment of LED position is another reason. In this experiment, we assume that this error is included with the total measurement error. The investigation of the force estimation method using the integrated pixel value in response to the different size of the measured object and the technique to decrease the measurement error will be our future work.

![Control points extracted from two tactile images obtained under different loading forces on the same object.](image)

**Fig. 11:** Control points extracted from two tactile images obtained under different loading forces on the same object. (a) Before pattern matching, (b) After pattern matching.

![Elastic modulus estimation of soft polymers, CL2000X and CL2003X.](image)

**Fig. 12:** Elastic modulus estimation of soft polymers, CL2000X and CL2003X.

**V. Discussion and Future Work**

In this paper, a high-resolution tactile imaging sensor capable of measuring the elastic modulus of the touched object was constructed. To verify the performance of the proposed sensor, we measured the elastic modulus of a soft polymer. The experiment showed that the proposed sensor determined the elastic modulus with less than 5.38% error. This is a preliminary result. To demonstrate the sensor’s capability to detect an elastic object embedded in a soft tissue, it is necessary to perform more experiments. Therefore, we will build a tissue phantom with inclusions. The phantom will consist of a silicone composite having a Young’s modulus of approximately 5–10 kPa [2]. The inclusions will consist of another silicone composite, the stiffness of which will be much greater (50–300 kPa) than the surrounding tissue phantom [2]. In the future work, experiments to measure the elastic modulus of inclusions in tissue phantoms will be performed.

The measured object must be smaller than the sensing probe area. If we are measuring the elasticity of a surface object, the contact area must be smaller than the probe area. However, if we measure a subsurface object, the probe area can be smaller than the contact tissue surface, as long as the inclusion is smaller than the probe area. In this case, the contact area, \( C_s \), is the area of the sensing probe with the known value. Thus, even if the hidden tumor exists in a variety of sizes and shapes, the contact area, \( C_s \), is always a constant, which is the probe area. One possible application of this sensor is the early breast cancer detection. The common breast tumor size is approximately 20 mm or smaller in stage 0 and I. In stage II, it is approximately 20 mm to 50 mm [35]. To detect the 50 mm breast tumor, the sensing probe should be bigger than 50 mm.

The background light disturbance and drifting of the light source are issues for the tactile sensor that operates based on the detection of light illumination. If the background light is too bright compared to the light of the tactile sensor, it causes background light disturbance. To prevent this, we use the tactile sensor in a relatively dark room. This way we minimize the background light disturbance. The drifting of light occurs if the sensor’s light source is positioned incorrectly and the camera captures the unnecessary light. In this case, the drifted light is shown as noise in the image. To prevent this, precise positioning and directionality of the light source is required. We carefully positioned and calibrated four light sources with acceptance angles calculated using Eq. (8). These two methods prevented the light disturbance and drifting of the light source issues during the preliminary experiments. In the future, we will also consider using the image processing techniques. More specifically, the Canny edge detector can be used to extract the boundary of the desired light scattering area in the image and remove the noise [36].
Typically, in the LED operation, approximately 20% input power is converted to light and 80% to heat [37]. Heat at the junction of LED affects the overall performance of the LED in terms of light output and spectrum. The amount of light emitted by the LED decreases as the junction temperature rises. Thus LED luminaries require precise power and heat management systems, since most of the electrical energy supplied to an LED is converted to heat rather than light. Without adequate thermal management, this heat can degrade the intensity of LED and finally affect the calibration result of the sensor. In the current sensor design, the LED is directly connected to the power source, resulting in the intensity drift caused by the temperature effect and the variation of the resistance in the power source. In our experiments, the sensor takes approximately 20 seconds to imaging the object. Thus the LED intensity drift caused by the temperature effect is not large. In the next sensor design, we will consider the thermal management system such as heat sinks to release heat from LED. The flip-chip package type LED will also be considered to reduce the thermal resistance of the LED. To prevent the variation of the resistance in the power source, we will also consider the constant current LED drive circuit.

Eq. (10) is material-dependent. Depending on the material, the relationship could be linear or nonlinear. In this paper, we assumed that the sensed material is homogeneous and isotropic. In this case, the relationship between the normal force and the integrated pixel value of the tactile image is linear. Thus, the applied force is estimated from the integrated pixel values using the normal force versus the integrated pixel values table, which is previously obtained by the calibration. Then the applied stress, which is the force per unit area, is obtained by dividing the applied force by the contact area. We have verified this approach in Section IV for the homogeneous and isotropic material. If the sensed material is inhomogeneous and anisotropic, Eq. (10) will not be valid. We will have to re-derive Eq. (10) for inhomogeneous and anisotropic material case.

The measurement range of the device is controlled by the elastic modulus of the sensing probe. This is determined by how we mix two components of Polydimethylsiloxane (PDMS), the viscous fluid silicone and the catalyst. The viscous fluid silicone is hardened by the catalyst. If the amount of catalyst increases, the elastic modulus of the PDMS increases. If the amount of catalyst decreases, the elastic modulus of the PDMS decreases. In this paper, the measurement range of the sensor designed for the soft polymer elastic modulus measurement is from a normal force of 0 mN to 2500 mN. In the future work, we will investigate the different measurement range of the device depending on the different elastic modulus of the sensing probe.

VI. CONCLUSIONS

In this paper, a novel tactile imaging sensor capable of measuring the elastic modulus of the contacted object is designed and experimentally evaluated. To emulate the human finger layer, a multi-layer optical waveguide was fabricated as the main sensing probe. Total internal reflection principle is used to obtain the high resolution tactile image. In order to obtain the elastic modulus of the compressed object, a new non-rigid pattern matching algorithm is proposed. The performance of the tactile imaging sensor is experimentally verified using the soft polymers. The results show that using the proposed sensor, the elastic modulus can be estimated within 5.38%.

REFERENCES


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