Effect of Unsaturated Flow on Steady Seepage

Michel C. Boufadel[1], ASCE member, Makram T. Suidan[2], ASCE member, and Albert D. Venosa[3]

Abstract

Steady seepage in two-dimensional domains is investigated using a dimensionless formulation for variably-saturated media that depends on three dimensionless parameters, M, n, and α. The parameter M is the product of the anisotropy ratio and the squared ratio of the vertical length scale to the horizontal length scale. The parameter n increases with the uniformity of the pore sizes, and α represents the ratio of the domain height to the height of the capillary fringe. For trapezoidal domains, the seepage face height decreased first with decreasing M and increased later at low values of M. The effect of α was maximal at extreme M values, while the effect of n was minor for all (M, α) values. Guidelines for scaling physical systems are provided.

Introduction

Seepage faces occur commonly in unconfined groundwater flows. Accurate estimation of the seepage face height is important in investigating the stability of porous structures (Freeze and Cherry, 1979). Equally important is the computation of the outflow (or discharge) from unconfined domains; outflow values are needed, for example, in the design of constructed wetlands. Boufadel et al., (1999) investigated steady seepage in 2-D rectangular and trapezoidal domains using a variably-saturated flow model (thus water flow occurs in both the saturated and the unsaturated zones of the porous domain). They demonstrated that the seepage face at the exit of rectangular domains computed by the variably-saturated model is always larger than that computed using a saturated-flow model. They also observed that the outflow from rectangular domains can be, in certain cases, 60% larger than the outflow computed using a saturated-flow model. The latter is equal to that obtained using the Dupuit assumption (Bear, 1972). The approach taken by Boufadel et al. (1999) relied on a dimensionless formulation for water flow in variably-saturated media developed by Boufadel et al. (1998).

----

[1] Dept. of Civ. and Envir. Eng., Univ. of Cincinnati, Cincinnati, Ohio. Now, Res. Asst. Prof., Dept. of Envir. Eng. and Sc., Clemson Univ., P.O. Box 340919, Clemson, SC, 29634-0919. Corresponding author; Phone: (864) 656-5574, Fax: (864) 656-0652, e-mail address: mboufad@ces.clemson.edu

[2] Prof., Dept. of Civ. and Envir. Eng., Univ. of Cincinnati, Cincinnati, OH 45221.

[3] Microbiologist U.S. Envir. Protection Agency, Nat. Risk Mgmt. Res. Lab., Cincinnati, OH 45269.

This work further elucidates the advantages of the dimensionless formulation of Boufadel et al. (1998) by discussing issues related to scaling of physical systems and by investigating steady seepage in an anisotropic trapezoidal domain simulating a dam.

Governing Equations

Neglecting the compressibility of water and the source/sink term, the governing equation for two-dimensional variably saturated flow in porous media written in a dimensionless form is (Boufadel *et al.*, 1998):

$$\phi\frac{\partial S}{\partial t} + S_s S\frac{\partial \psi}{\partial t} = M\frac{\partial}{\partial x}\left(K_x\frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_z[\frac{\partial \psi}{\partial z} + 1]\right) \quad (1)$$

where M is a dimensionless number given by:

$$M = \frac{K_{xo}}{K_{zo}}\frac{L_z^2}{L_x^2} \quad (2)$$

and where $K_x = K_x^*/K_{xo}$, $K_z = K_z^*/K_{zo}$, $x = x^*/L_x$, $z = z^*/L_z$, $t = t^*/(L_z/K_{zo})$, $S_s=S_s^*L_z$, $\psi=\psi^*/L_z$, and where the (*) stands for dimensional quantity. The term ϕ is the porosity[-] (where - signifies dimensionless quantity), S[-] is the soil moisture ratio (or relative soil moisture) given by $S=\theta/\phi$ where θ is the water content [-], ψ^* is pressure head [L]; S_s^* [L^{-1}] is the specific storage per unit water weight, given by $S_s^*=\delta\phi/\delta\psi^*$, and K^*_x, and K^*_z are the horizontal and vertical hydraulic conductivities, respectively, and they are assumed to be parallel to the major axes of anisotropy. The subscript "o" represents the saturated hydraulic conductivities.

The soil moisture and the hydraulic conductivity are correlated experimentally to pressure head by the van Genuchten model (1980) which is written in a dimensionless form as (Boufadel *et al.*, 1998):

for $\psi \geq 0.0$: $K_x=K_z=1.0$ **(3a)**

For $\psi < 0.0$:

$$S_e = \frac{S - S_r}{1 - S_r} = \left[\frac{1}{1 + (\alpha|\psi|)^n}\right]^m \quad (3b)$$

and

$$K_j = S_e^{(1/2)}\,[1 - (1 - S_e^{1/m})^m]^2 \quad (3c)$$

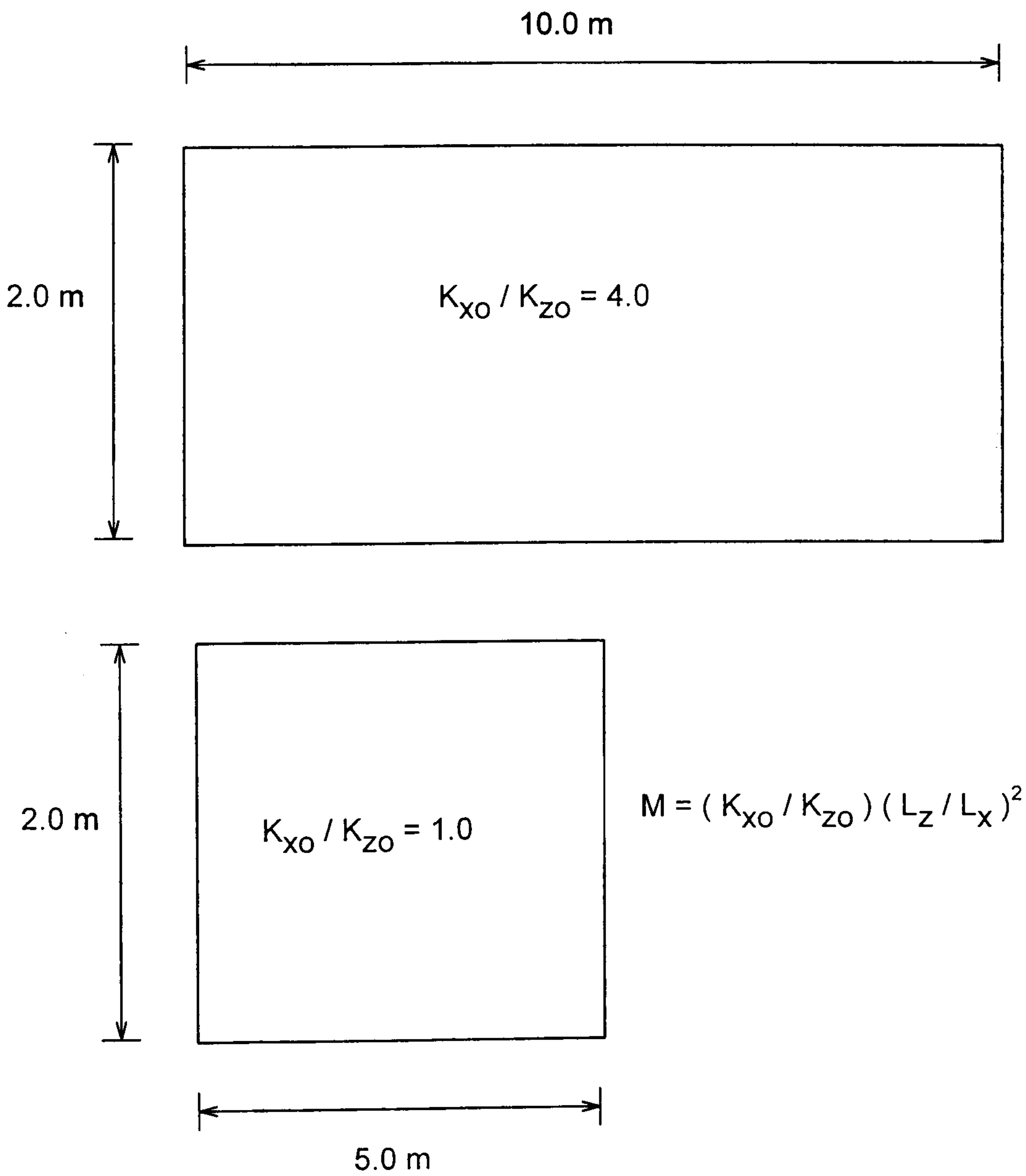

Figure 1: Transforming an Anisotropic Domain (Top) to an Isotropic Domain (Bottom). Correct Transformation Requires Conservation of M.

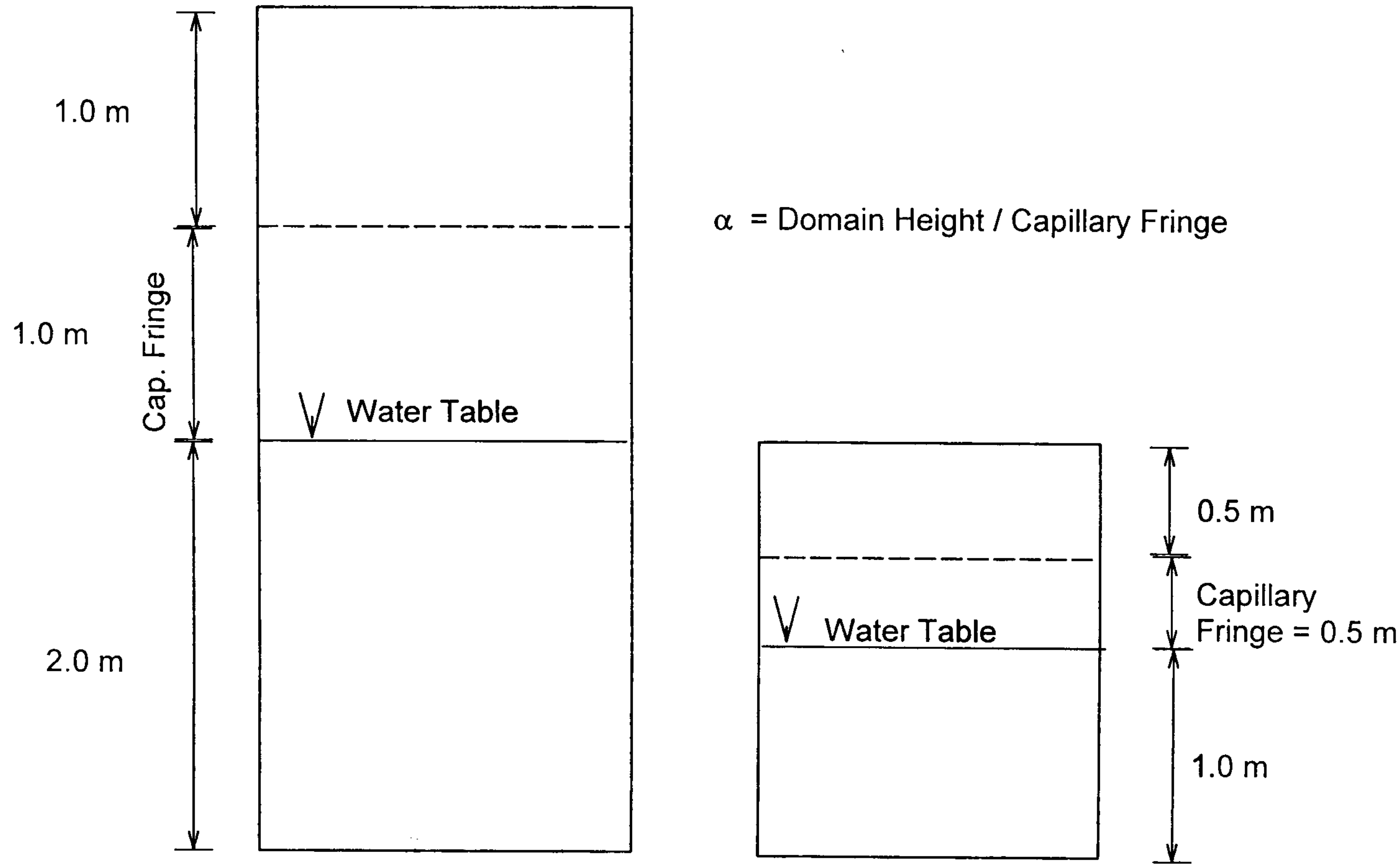

Figure 2: Scaling-down the Vertical Length of Porous Media. Correct Scaling Requires Conservation of α.

where j= (x, z); m=1-1/n; and $\alpha = \alpha^* L_z$. The parameter α^* $[L^{-1}]$ represents a characteristic pore size and higher α^* values imply a coarser material. The inverse of α^* provides an estimate of the capillary fringe (zone of considerable moisture) (Bear, 1972). The term n represents the uniformity of the porous medium with higher values of n implying a more uniform pore space distribution (van Genuchten, 1980).

Under steady state conditions, all the terms associated with the temporal derivative in (1) vanish; the effective saturation ratio is computed directly from the second equality in (3b), which depends solely on n, α, and ψ. Hence, ϕ, S_s, and S_r values need not be considered in the remainder of this manuscript. Only M, n, and α are needed. The dimensionless number M combines the effects of anisotropy and the aspect ratio of the porous medium. The dimensionless number α represents the ratio of the domain height, L_z, to the thickness of the capillary fringe, $1/\alpha^*$.

Equations (1), (2), and (3) provide guidelines for scaling (down or up) physical system. For example, if it is desired to build an isotropic physical model to simulate an isotropic or anisotropic prototype (for example a dam or a trench), it suffices to use an isotropic soil such that M, n, and α are conserved between the prototype (i.e., large scale) and the physical model. Boufadel et al. (1998, 1999) observed that the effects of n on the seepage face height and water flow in the porous domain are minor in comparison to those of M and/or α. Therefore, for most practical applications it suffices to conserve M and α. Figures 1 and 2 illustrate the scaling methodology. Figure 1 considers a case where an anisotropic domain is "transformed" into an isotropic domain. It shows that the resulting isotropic domain is smaller than the anisotropic prototype, which is advantageous from a logistic point of view. Of course, the horizontal contraction is limited by the stability of the slope in the laboratory physical model. Figure 2 illustrates the vertical contraction. It shows that the capillary fringe in the physical model has to be smaller than that of the prototype. This implies that a coarser material has to used in the physical model in comparison with the prototype. The reader is referred to the work of Boufadel et al. (1998) for further discussion and for guidelines for scaling transient regimes.

Steady State Seepage Through a Dam

Under a steady state regime, failures of an earth dam can result from excessive seepage, from piping at the toe, or from slope failures on the dam faces. Many textbooks provide design features to reduce the probability of failure (Cedergren, 1967). In most cases, the solution results in a reduction of the seepage face height by lowering the highest exit point at the downstream face of the dam (Freeze and Cherry, 1979). For this reason, a nomograph is presented herein for seepage face height for a typical dam on an impermeable stratum as shown in Figure 3. The geometry of the domain was selected to represent average sized dams because the crest width is 20% of the base of the dam, which would be large for larger dams (Cedergren, 1967). Figure 4 shows seepage face heights for a wide range of M. The large values of M do not necessarily result from the aspect ratio L_z/L_x, which is dictated by stability requirements; they can result from anisotropy due to compaction (Cedergren, 1967). The variation of the seepage face height with α, n, and large M is similar to the rectangular section. However, the seepage face height increased at smaller values of M, which is due to the sloping downstream face of the dam. This is because small values of M result in water flowing more freely in the horizontal direction than downward, towards the narrow end of the dam. The value $M \approx 0.05$ provided the most economical solution because it resulted in the smallest seepage height. A complete set of nomographs for dams of various geometry is not presented here for brevity.

Summary and Conclusions

Seepage problems were investigated using a dimensionless formulation for water flow in two-dimensional variably-saturated anisotropic media. The dimensionless formulation depended on three dimensionless parameters, M, n, and α. M is the product of the anisotropy ratio and the squared ratio of the vertical length scale to the horizontal length scale. The parameter n represents the pore size distribution and α represents the ratio of the vertical length of the domain to the height of capillary fringe. The dimensionless formulation shows that scaling-down physical system requires a coarser porous medium in the small-scale physical model in comparison with the porous medium in the large-scale porous medium. For trapezoidal domains, the seepage face height decreased first with decreasing M and increased later at low values of M. This allowed the selection of the most economical aspect ratio of a hypothetical dam.

Acknowledgment

This work was supported, in part, by the US Environmental Protection Agency's National Risk Management Research Laboratory, Cincinnati, Ohio, under Cooperative Agreement No CR-821029. However, it does not necessarily reflect the views of the Agency, and no official endorsement should be inferred.

References

Bear J. (1972). Dynamics of fluids in porous media, American Elsevier, New York, NY.

Boufadel, M. C., Suidan M. T., Venosa A. D., and M. T. Bowers (1999). Steady seepage in trenches and dams: Effect of capillary flow, *J. of Hydraulic. Eng.*, ASCE, v 125, March issue.

Boufadel, M. C., Suidan M. T., Venosa A. D., Rauch C. H., and Biswas P. (1998). 2-D variably saturated flows: Physical Scaling and Bayesian estimation *J. of Hydrol. Eng.*, ASCE, v 3, 223-231.

Cedergren, H. R. (1967). Seepage, Drainage, and Flow Nets, John Wiley and Sons, NY, p 489.

Freeze, R. A., and Cherry J. A. (1979). Groundwater, Prentice Hall, NJ, p 604.

van Genuchten, M. Th. (1980). A closed-form equation for preciting the hydraulic conductivity of unsaturated soils, Soil Sci. Soc. Am. J., 44, 892-898.

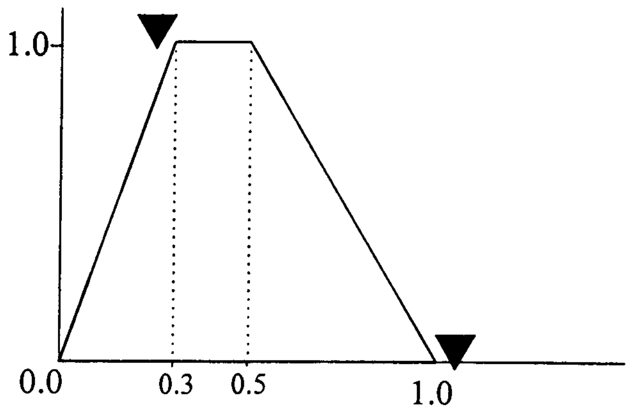

Figure 3: Geometry and Boundary Conditions of Trapezoidal Dam

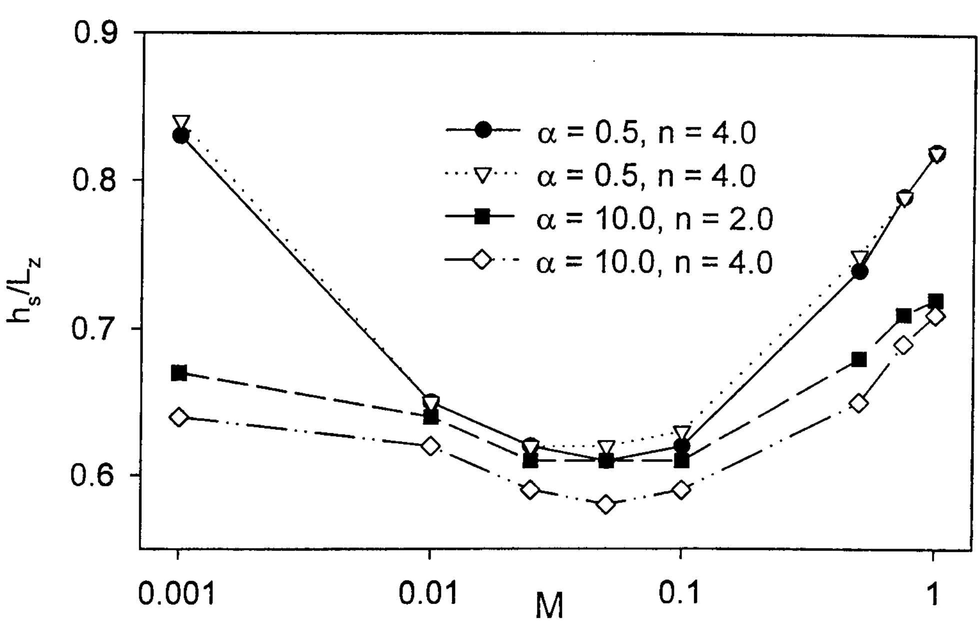

Figure 4: Dimensionless Seepage Face Height For Various Values of M, α, and n, (Boufadel et al.,1999).