Proposal

for a

Master of Science Program

in

Integrative Applied Mathematics

Submitted by

The Department of Mathematics
College of Science and Technology
Temple University

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Contents

1 Introduction

This proposal is for a new Master's program at Temple, a Master of Science degree in *Integrative Applied Mathematics*. The core subjects of the degree are mathematical in nature and motivated by the needs of science and industry. The term "integrative" reflects the essential non-traditional mixing of topics often taught in different departments or even schools.

2 Rationale for a Master's Program in Integrative Mathematics

The current decade, the last of this century, is witnessing an economic and technological boom comparable to that of the 1920's. The earlier boom was fueled by the electrification of America and the world (and the end of the great war): Electricity was changing every machine, every factory, every home, and every business. This led to the phenomenal growth of the stock market in the post-war years. The current boom is fueled by the computerization of America and the world (and the end of the cold war): Silicon chips are changing the nature of communication, business, education and technology, even at the most minute level. Of course, this has bee the basis of the 1990's long bull market. Today a corporation — if it so chooses — can be administered on a totally paperless basis, with web-based and on-demand communication with suppliers and/or customers, eliminating inventory and other costs. One of the most successful such companies is *Dell Computer*, whose market capitalization has multiplied 100-fold in a few short years. Less known is the role that mathematics plays in these sweeping societal changes. Very few people today know of the sophisticated mathematics algorithms in chip design, image compression, robotics, internet security, or finance. The reason is that many of these issues are only taken up in more advanced courses in graduate school, often at the Ph.D. level and in a piecemeal fashion.

Like many other big cities, the western suburbs of Philadelphia (the rte. 202 corridor, Malvern, and Chester county) are burgeoning with high-tech companies. These range from computer components manufacturers to aerospace software designers, to health care and medical software, to pharmaceuticals, and to the local investment firms that feed off this growth. Employees of such companies, armed with a bachelor's degree in business, science or engineering, are often surpised to find the need for additional training in *mathematics*, something they thought they've finished with. This training is difficult to find, as traditional undergraduate mathematics programs are not specialized enough while graduate programs involve more of a time investment and typically involve highly specialized training aimed at academic professions or national laboratories.

An integrative mathematics program would have three goals: To prepare students in the core essentials necessary, to train them in the applied area of their choice (e.g. risk management or tomography), and to enable them work through a project or internship in the area of their choice. There are very few such programs in the US today. In the Delaware valley, surprisingly, there are none. Nationally, we are aware of similar programs at Carnegie-Mellon University, Columbia University, Georgia Institute of Technology, and

New York University.

An integrative mathematics Master's program — a "consumer" Master's — tailored to the needs of such working professionals, or more generally to any post-baccalaureate students who want to work in technology and/or finance, would be met with strong demand. It would draw not only from the greater Philadelphia area, but also from overseas, as foreign governments increase their support of interational graduate studies for their students, future educators and professionals.

> "We shall need ... the creation of a new breed of mathematical professionals able to mediate between pure mathematics and applied science. The cross-fertiliization of ideas is crucial for the health of ... science and mathematics."

M. Gromov, Notices of the American Mathematical Society, August 1998.

3 The Integrative Mathematics Program at Temple University

"To disseminate integrative mathematical methods linking science, industry, and information technology."

The mission of the proposed program is to provide graduate students (post-bachelor's) and working professionals with the mathematics background necessary to solve industrial problems and to think conceptually, creatively and abstractly in dealing with business and technological issues. Graduates of this program will have a well-developed sense of how mathematics can help them in their work environment and, at the same time, make them more attractive to industrial companies or government agencies looking to hire. An optional feature of the program is its internship, which will provide the student with a natural bridge between the program courses and the outside world.

There are three reasons for setting up such a program at Temple. The first is the lack of such a program in the Delaware valley, the second is the burgeoning growth of high-tech industries in Chester County and the consequent high demand for technologically-trained individuals, and the third is that such a program is a reflection of national trends.

We emphasize that much of the demand for such individuals is for "knowledge technology", as opposed to "hardware technology". Perhaps this is because the supply of trained "hardware technologists" is close to satisfying demand, while "knowledge technologists" are in extremely short supply. Indeed, recently a number of technology CEO's (including Microsoft's Gates) lobbied congress to raise the annual limit on the number of "technology" immigrants.

> "Recipients of master's degrees from the School of Mathematics are greatly in demand."
>
> *Georgia Institute of Technology web site:*
> *http://www.math.gatech.edu/som/graduate-study.html#Placement.*
>
> "A new profession demands a new program."
>
> *University of Toronto, Mathematical Finance brochure.*

3.1 Centers of Excellence and the Integrative Program

Current strategic planning both in the College of Science and Technology and in the University as a whole indicates that new, multidisciplinary centers of excellence will be created on campus in the near-term. The program will be ideally positioned to benefit from and enhance such centers. At the same time, the program does not *depend* on these centers for success.

There are several reasons why such centers will be helpful to the program. First, they would provide added cachet to the program, both externally and internally. Externally, in advertising, cooperation with centers would make clearer Temple's commitment to applied areas and emerging technologies. Internally, it could be a credential added to any faculty teaching one of the program's courses and, thereby, cooperating with a center of excellence. This would also provide a focal point for future developments, such as grants, conferences, visitors, and additional degree programs. In addition, some of the involved faculty may come from physics, computer science, economics, electrical engineering, or finance. Because of this, it makes the most sense to build inter-departmental cooperation into the program.

Moreover, linking up with centers will facilitate possible differential tuition for programs relevant to such centers. Initially, however, we expect the program to involve little overhead costs as the program will be run entirely by the Department of Mathematics.

We anticipate that direct links will be placed in the University's web portal to the forthcoming centers' web sites to facilitate and attract attention. Such links will also benefit this program. We plan to use technology as much as possible, including web-based teaching/learning and an electronic web-based application process.

4 The Curriculum

The program is divided into three groups: The foundational mathematics courses, the "applied" concentration courses, and the optional internship. We emphasize that the foundational courses will discuss applications, and the applied courses will discuss mathematics, so the division will not be sharp, and the internship is optional, depending on the student's interests.

The concentration topics are mathematics courses; as such, they will differ substantially from similar courses offered outside the mathematics department. For example, we have been encouraged by the risk management department at Temple to run the mathematics of finance courses, as they are not offered in that department and they provide students with additional problem-solving skills. For most of the students entering the program, the key aspect of their background will be the level of their mathematical ability and general intellectual maturity, rather than specific skills in, say, economics or engineering.

The program will initially offer six concentrations, of which the student chooses two or three, depending on their emphasis. There will also be an opportunity for *per-concentration certification* upon the completion of individual concentrations together with certain foundational courses (see section 4.2). For such students, the internship is replaced by an additional concentration.

A concern is that students choosing not to take an intenship may end up with a weaker program. In fact, the emphasis on focused concentrations forces the students to learn the topics in depth, so that these students actually end up with a more rigorous degree.

Although all the courses are at the graduate level, the courses will be set up to assume a minimum of background knowledge (admission requirements are listed below). We do this because we expect students to come from other science, business, or engineering backgrounds. Nevertheless, we do assume that the students have the maturity and desire to expend the energy necessary to work through such a program. One emphasis will be on honing abstract thinking tools, as high-tech companies often hire applicants not on the basis of what they know but on what they can learn.

The courses will be taught within the Department of Mathematics, but, partially because of a potential differential tuition, and partially because of such a program's additional cachet, we are hoping to run future installments of some of these courses under the rubric of the emerging Temple Centers of Excellence, once those are created (see previous section). We emphasize that the structure of the program does not assume the existence of the centers but does include built-in cooperation with them, once they surface.

All courses will carry three credits and will be at the graduate level. There will be six applied concentrations, each consisting of two courses. Initially there will be no electives. For a full degree, students must take 36 credits, consisting of a combination of foundational courses, concentration courses, and an (optional) internship. The internship may be 3 or 6 credits.

4.1 Admissions requirments

Applicants must hold a bachelor's degree, or its equivalent, along with an undergraduate mathematics background which includes multivariable calculus as well a concentration of at least 16 credit hours of upper level undergraduate courses in one or more of the areas of mathematics, business and finance, science, computer and engineering technology.

4.2 Degree and Certificate Options

A student may enroll in the full degree program, or obtain a *certificate* upon the completion of a set amount of work. This latter option may be attractive to students whose work schedules force them to work on this degree on a part-time basis. Completion of three of the certificates is considered equivalent to completing the requirements for the full degree. A concentration consists of two 3-credit courses.

The time limit for degree completion is 5 years. Any exceptions to this must be taken up in consultation with the Graduate School.

Certificates will be named and awarded according to Graduate School guidelines.

Certificate Option

- 2 foundational courses ($2 \times 3 = 6$ credits),
- 1 concentration ($2 \times 3 = 6$ credits),

Full Degree — Option 1

- 6 foundational courses ($6 \times 3 = 18$ credits),
- 2 concentrations ($2 \times 2 \times 3 = 12$ credits),
- 2 internships ($2 \times 3 = 6$ credits).

Full Degree — Option 2

- 5 foundational courses ($5 \times 3 = 15$ credits),
- 3 concentrations ($2 \times 3 \times 3 = 18$ credits),
- 1 internship (3 credits).

Full Degree — Option 3

- 6 foundational courses ($6 \times 3 = 18$ credits),
- 3 concentrations ($2 \times 3 \times 3 = 18$ credits),

Full Degree — Option 4

- 3 certificates ($12 \times 3 = 36$ credits),

4.3 The Foundational Courses

- Analysis I
- Analysis II
- Linear Algebra
- Differential Equations I
- Differential Equations II
- Probability

These courses are described in detail in section 9. Actual course numbers are in section 7. Later more courses may be added, but for now the courses cover

- one-variable calculus systematically,
- computational aspects of transform theory and multi-variable calculus,
- a systematic introduction to linear algebra and matrices,
- ODE's: linear systems, elements of nonlinear theory, linear stochastic systems,
- PDE's: Linear and nonlinear equations, computational skills,
- random variables in one and several dimensions, gaussians, computations.

4.4 The Concentrations

Each concentration consists of two 3-credit courses.

- **Imaging Concentration:** Tomography and Integral Geometry, Wavelets and Compression.
- **Control Concentration:** System theory and Optimal Control, Estimation and Filtering.
- **Finance Concentration:** Mathematics of Finance I and II.
- **Symbolic computation concentration:** Symbolic computation, mathematical software, cryptography.
- **Numerical Analysis concentration:** Numerical Techniques, parallel programming, nonlinear PDE's.
- **Stochastic Concentration:** Stochastic calculus, Statistical Modelling.

For example, a student would take 5 foundational courses, concentrations in imaging, finance, and symbolic computing, and 1 3-credit internship.

4.5 The Internship

Students participating in this program may choose to take a 3- or 6-credit internship, consisting of work carried out in an industrial or business setting. For students who come to us from a working environment, this setting could be their current one, simplifying logistical considerations. However, to keep the program flexible, students may also choose to complete the program in a non-internship *certificate* mode (section 4.2).

It is expected that the director of the program will oversee all aspects of each student's internship, in consultation with the departmental graduate committee, from placement to evaluation to support and analysis.

The student will join as an intern in a particular department or group in participating firms, the idea being that the student will pick up valuable day-to-day experiences, while the firms, in turn, will be able to "try out" our students without committment or financial outlay, and moreover train them to their specifications.

The technology companies we approached were genuinely excited upon hearing of this program at Temple and asked us to send them our graduates.

Currently we have committments from four companies:

- **Susquehanna Investments, Bala Cynwyd, PA**. The contact person is Dr. Doug Costa, Head of Quantitative Research. This is a large investment house (they trade 5% of the NASDAQ daily volume) that uses sophisticated mathematical models to trade in the markets.

- **Innovative Software Solutions, Cherry Hill, NJ.** The contact person is Steve Webb, Vice President. This is a software company specializing in benefit fund administration.

- **Optimum Capital Management — Wayne, PA.** The contact person is Nabil Estefan, President. This is a small hedge fund management company investing in U.S. equity markets. Mr. Estefan said explicitly he could use a staff member who is good at modelling.

- **Systems and Computer Technology, Malvern, PA** (tentative). The contact person is Michael Peters, Marketing. This is a large software company specializing in administrative software for industry and higher education (Temple is a customer).

We emphasize that this is just a beginning. Once we get preliminary approval, we will start systematic canvassing, e.g., send out a letter/brochure to each of the many technology firms in the Malvern Great Valley Corporate Center.

5 Administration and Faculty Resources

5.1 The Program Director

The program is administered by the department of mathematics; as such, the departmental executive committee will be considered this program's steering committee. In particular, this committee is responsible for appointing and/or change the program director, addressing student grievances, and providing program oversight.

The director of the program will have the following duties, performed in consultation with an advisory committee to be appointed by the department's executive committee. Day-to-day issues will be carried out in consultation with the department's graduate committee.

Coordination of the recruitment and admissions process

This includes review of student applications, students' background and suitability for the program, including the students' likelihood of completing the program. This last aspect is especially important to the corporations sponsoring the students. The Director will also work with College faculty to establish regular and effective recruitment activities.

Orientation

The Director will provide initial guidance and assign an advisor to each incoming student, and integrate program orientation with that of the college. The role of the advisor will be similar to that of the current departmental undergraduate advisors.

Scheduling and Staffing

The Director will establish a program course schedule which fits both department course schedules and student and market demands such as night classes. The director will select course instructors from the mathematics department faculty, and in special instances, outside faculty.

Internship: Deployment, Oversight, and Administration

The Director will initiate internship relations with outside companies and institutions and select a faculty adviser for each internship project. In particular, the Director must insure that quality control and effective grading are maintained at a graduate level. Grading of the internship will typically be based on working through a project involving both mathematics and the work/corporate setting as well as completing an intership "paper" carefully describing all aspects of the project.

Progress Tracking

The director will ensure that students maintain an effective rate of progress and direction so as to complete the program in a reasonable time period and with a balanced course portfolio.

This aspect is especially important to sponsoring corporations, and will encourage "repeat customers".

Trend Analysis And External Funding

The director will promote the program in the academic and industrial communities and seek innovative sources of external funding in order to farther develop and enhance the program.

5.2 Faculty Resources

Most of the faculty teaching the program courses will come initially from the Department of Mathematics. Some courses may be taught by faculty in electrical engineering, finance, etc. Later on, as the program expands, we may bring in outside adjuncts for more advanced courses. We emphasize, however, that the Department of Mathematics faculty is fully capable of designing and teaching a world-class program in the subjects chosen. Please see the course descriptions. In later years, as the program expands, new faculty lines may be generated. In the best case scenario, these may be partially funded by industry.

Courses in the program will typically involve more work, effort, and creativity on the part of the instructor, partly because of their integrative nature, and partly because they will be writing-intensive. By this we mean that the program expects that a substantial portion of every course grade to reflect the student's writing/communication ability, and that in each course writing assignments be handed out early and frequently. *We are doing this simply because we have heard from employers repeatedly that new hires in technology generally have poor writing skills.* Written guidelines, set by the department's graduate committee, will be provided to each course instructor so as to ensure uniformity across the program's course offering.

6 Computing and Library Resources

Initially the program will rely mostly on the Department of Mathematics department computing and library resources.

A thorough website for the program will be developed in conjuction with the mathematics department. The Department of Mathematics webmaster has agreed to participate in this effort. The advantages of this are additional exposure of the program and more streamlined administration.

We ask that a direct link to this program be placed in the university's web portal, to facilitate and attract attention.

7 Course Structure

7.1 Program Courses

Traditionally, the Mathematics courses at the 400 level served students who were admitted to the Ph.D. program but whose background was deemed not sufficiently strong for the 500 level sequence. The numerology and sequencing of the 400 level offerings reflects, in part, historical developments in the program and is no longer as logical and intuitive as it could be. With the introduction of this program the 400 level courses will also serve a new population of students with new objectives, and will promote the *integrative* theme. For these reasons we feel that the time is ripe for a revision of the 400 level sequence as a whole. This will include the addition of this program's innovations and their integration with the basic courses that serve to prepare for our traditional Ph.D. curriculum.

Here is the list of program courses; these are currently on the books, but the numbering system is somewhat confused after years of haphazard course additions/deletions. Because of this, we hope to renumber our 400 level courses at some future date.

Course Title	Current Number	Possible Future Number
Analysis I	417	401
Analysis II	418	402
ODE	462	421
PDE	451	422
Linear Algebra	477	411
Probability	433	412
Imaging I	633	431
Imaging II	634	432
Math Finance I	573	441
Math Finance II	574	442
System Theory & Control	415	451
Estimation & Filtering	697	452
Statistical Modelling	434	461
Stochastic Calculus	698	462
Symbolic Computing	563	471
Cryptography & Number Theory	471	472
Computing for Mathematicians	472	473
Numerical Analysis I	404	481
Numerical Analysis II	414	482

The contents of these courses on the books agree with these topics. What is new and different is the emphasis on integrative skills.

8 Course Descriptions

Note: Some course decriptions contain much more than can be covered in one semester, and will be modified during the first year. All courses in this proposal are at the 400 level.

8.1 Analysis I

Goals

This course is a systematic high-level one-variable calculus course.

Prerequisites

Some previous calculus exposure.

References

- O.Hijab, Introduction to calculus and classical analysis, Springer (1997).
- A.E. Taylor, Advanced calculus .
- J.Marsden, Elementary classical analysis.

Core Topics

- The real number system: Completeness, limits, sequences and series.
- Continuity and differentiability.
- Integration: Area and the fundamental theorem of calculus.
- Some applications.

8.2 Analysis II: methods of applied mathematics.

Course Goals

This course involves multi-variable calculus and transform theory with computational aspects emphasized. The concepts and techniques are likely to be useful in virtually all other courses in the *integrative* program.

Course Prerequisites

Analysis I.

References

- O.Hijab, Introduction to calculus and classical analysis, Springer (1997)
- R.Strichartz, The way of analysis, Jones and Bartlett (1995)
- A.E. Taylor, Advanced calculus
- J.Marsden, Elementary classical analysis

Core Topics

- Topology and Geometry of Euclidean space.
- Continuous functions and their properties on compact sets.
- The differential: local extrema, Taylor's theorem, the inverse and implicit function theorems.
- Curves, surfaces and manifolds.
- Multiple integrals: change of variables, divergence theorem, spherical integration
- Integral transforms: examples
- Fourier transforms, distributions, notion of singularity
- Special functions: Beta, Bessel, Gamma, Orthogonal polynomials

8.3 Differential Equations I

Goals

This course is an introduction to linear ordinary differential equations (ODE's), nonlinear theory, and linear stochastic ODE's.

Prerequisites

Some previous calculus exposure, Linear Algebra concurrently.

References

- Hoel, Port and Stone, Introduction to Stochastic Processes.
- Hirsh and Smale, ODE's and Linear Algebra.

Core Topics

- Linear ODE's with constant coefficients, Euler's theorem.
- Linear systems, stability and asymptotic behavior.
- Elementary nonlinear theory.
- Linear stochastic systems.

8.4 Differential Equations II

Goals

This is a basic course on partial differential equations (PDE's). Many of the fudamental processes in nature are described by partial differential equations. Examples include the flow of fluids,the diffusion of chemicals, the diffusion of chemicals, vibrations of solids, heat conduction,and the propagation of various types of waves. The course is intended to provide a useful and classical introduction to the subject, including first order equations and the linear second order equations which arise in science: the wave equation, the Laplace equation,and the heat equation. The equations are formulated and derived from physical principles.

Prerequisites

Analysis II, Linear Algebra, and Differential Equations I.

References

- E. C. Zachmanoglou and Dale W. Thoe, Introduction to partial differential equations with applications, Dover (1986).
- R. Haberman, Elementary applied partial differential equations, Prentice-Hall (1987).
- F. John, Partial differential equations, Springer (1982).
- R. Courant and D.Hilbert, Methods of mathematical physics.
- M. Pinsky, Partial Differential Equations.

Core Topics

- First order equations: Conservation laws and shocks.
- Laplace's equation, heat equation, the wave equation.
- Classification of PDE's: Elliptic, parabolic and hyperbolic PDE's.
- Fourier methods and orthogonal expansions

8.5 Linear Algebra

Goals

Linear Algebra, the study of lines, planes, etc., in higher-dimensional space, forms the basis of multi-variable calculus, differential equations, and multi-variate probability and statistics. This course is an introduction to linear algebra, both theoretical and computational.

Prerequisites

Some previous calculus exposure.

References

- Linear Algebra Done Right, S. Axler (Springer).
- Introduction to Linear Algebra, G. Strang (Wellesley-Cambridge Press, 1998)

Core Topics

- Vector spaces, linear transformations, dot products, basic results.
- Solution of linear systems by elimination, row reduction, matrix operations.
- Independence, Bases, rank, nullity, Orthogonality, projections, Gram-Schmidt process.
- Eigenvalues, diagonalizability, pseudoinverse, applications to differential equations.
- symmetric, positive definite, similar matrices. Singular value decomposition.
- Applications to computer graphics, Fourier series, economics.
- Δ on polynomials: spherical harmonics.
- Numerical linear algebra, condition numbers, iterative methods.
- Hermitian and Unitary transformations. The Fast Fourier Transform.

8.6 Probability

Goals

Probability is important as a cornerstone of signal processing and mathematical finance. The main goal of the course is to familiarize the student with the concepts, tools, and techniques commonly used in probability. Thus, students will not only learn the fundamental results and constructions of the theory, but will also learn to synthesize these results on their own and to produce similar results using the tools learned. Students are expected to understand and practice results well enough to become fluent in computations.

Prerequisites

Linear Algebra, Analysis II at least concurrently.

References

- Gray Davisson, Random Processes, Prentice-Hall, 1986.
- Hoel, Port Stone, Introd. to Stochastic Processes, 1972.
- Papoulis, Probability, Random Variables and Stochastic Processes, 3rd ed., McGraw-Hill, 1991.
- Karlin Taylor, A First Course in Stochastic Processes, Academic Press, 1975.
- Ross, Stochastic Processes, Wiley (1983).
- Lamperti, Probability.

Core Topics

- Real and multi-dimensional random variables, transformations, expectation and moments.
- Conditional probability, independence, conditional expectation.
- random processes, covariance and spectral density, Gaussian, Brownian, and Poisson types.

8.7 Numerical Analysis I and II

Goals

These two courses contain the basic information someone needs to get started with scientific computing; the basic mathematics, the most important computational methods (also called algorithms or recipes), and the practicalities of using a computer. Each of these is essential if one wants to reliably get the right answer. We have selected and shaped the material here on the basis of our experience teaching and doing scientific computing, and also on the basis of many interactions with people, from universities, government labs, and companies.

Prerequisites

Knowledge of calculus, linear algebra and basic probability. Some experience in programming.

References

- Numerical Recipes in Fortran or in C, Cambridge University Press
- J.M. Hammersley, D.C. Handscomb: Monte Carlo Methods, Methuen, 1965
- M.T. Heath, Scientific Computing, An Introductory Survey, McGraw-Hill, 1997
- D.G. Luenberger, Linear and nonlinear programming, Addison-Wesley, 1984

Core Topics

- Basic numerical ideas and programming practice: sources of error, conditioning and stability, computer arithmetic, numerical resolution, order of accuracy, accuracy check, etc
- Basic Monte-Carlo simulation: random number generators, Box-Muller methods for normals, Brownian motion simulation and application to finance, variance reduction, importance sampling, rejection methods, etc
- Fast Fourier transform and applications: fast Fourier algorithm, Nyquist frequency, sampling theorem, convolution and deconvolution, Wiener filtering, image compression, wavelet transformation, etc
- Nonlinear systems and continuation method,
- Optimization: constrained and unconstrained optimization, Newton's method, line searches quasi-Newton methods (BFGS, DFP)
- Sparse matrices, direct and iterative methods, degree reordering, nested dissection, conjugate gradient, preconditioning (SSOR, ILU), GMRES, etc

- Finite element methods, variational formulation, assembly of stiffness matrix, ideas of adaptive algorithms
- Numerical Methods for Time Dependent PDEs explicit and implicit finite difference schemes, basic facts about stability and convergence
- Introduction to Parallel Computing types of parallel computers and parallel programming models distributed memory and message passing, cost of communication data parallel programming and HPF (High Performance Fortran)

8.8 Tomography and Integral Geometry: Imaging I

Goals

This course introduces mathematical foundations of tomographic reconstruction and processing techniques that are increasingly found in major areas of science and technology. While the emphasis is on the basic abstract formulation, numerical considerations are also considered. Interested students may complete projects which form a closer link to practical applications in industry and science.

Prerequisites

Linear Algebra, Analysis II.

References

- Mathematics of Computed Tomography, F. Natterer, John Wiley & Sons (1986).
- The Radon Transform and Local Tomography, A.G. Ramm & A.I. Katsevich, CRC Press (1996).
- The Radon Transform, S. Helgason, Birkhauser (1980).
- Image reconstruction from projections: The Fundamentals of Computerized Tomography, G.T. Herman, Academic Press, (1980).
- The Radon Transform and some of its Applications, S.R. Deans, Wiley, (1983).
- Geometric Tomography, R.J. Gardner, Cambridge University Press (1996).
- Computed Tomography–physical principles. clinical applications and quality control, W.B. Saunders Co., Philadelphia (1994).

Core Topics

- The basic problem of computed tomography.
- The geometry of lines and planes in space.
- The Radon Transform and its relatives.
- Uniquenessand non-uniqueness: examples, counterexamples, support theorems.
- Inversion formulas.
- Estimates, stability, range conditions, attenuation.
- Sampling theory.

- Singular value decompositions, stability, ill-posedness.
- Numerical reconstruction methods.
- Limited angle tomography, interior and exterior problems.
- Distributions, singular support, microlocalization.
- Wavelets in the abstract.
- Local tomographic reconstruction using wavelets.
- Incomplete data problems.

8.9 Wavelets and Compression: Imaging II

Goals

This course introduces mathematical foundations of wavelet expansions and their applications in imaging science. These alternatives to Fourier analysis are finding increasing popularity in current applications.

Prerequisite

Linear Algebra, Analysis II.

References

- A Friendly Guide to Wavelets, Gerald Kaiser Birkhauser-Boston, 1994 (fifth printing, 1997).
- An introduction to wavelets, Charles K. Chui, Academic Press, (1992).
- Fundamentals of Digital Image Processing, A. K. Jain, Prentice Hall (1989).

Core Topics

- Multiresolution analysis–the concept.
- Filter banks, coding.
- Continuous time analysis and synthesis.
- Image Quantization.
- Fourier transforms: continuous and discrete; convolutions.
- Smoothing, edge detection, sharpening, filtering, color representation, sampling.
- Lossless image compression, lossy image compression.
- Image geometry.
- Advanced topics: Physical wavelets, electromagnetic, radar, scattering, acousting.

8.10 Mathematics of Finance I and II

Goals

This year-long course course is an introduction to finance, with an emphasis on the mathematics. Although starting from scratch, the course will move at a mature pace, and will be profitable even for students coming from an undergraduate degree in finance. This course is also suitable for professionals who want to acquire a working knowledge of how derivatives can be analyzed. The main text (Hull) contains a large amount of material and exercises. Grading will be based on homework exercise sets, exams, and an end-of-term paper.

Prerequisites

Some previous calculus exposure. Previous knowledge of finance is helpful but is not assumed.

References

- Hull, Futures, Options, and Other Derivative Securities, third edition.
- Karatzas and Shreve, Mathematical Finance.
- S. Shreve, Lectures on Mathematical Finance
- Duffie, Dynamic Asset Pricing Theory, Princeton University Press.

Core Topics

- Basic orientation, markets, hedgers, speculators, arbitrageurs.
- Forward and future contracts, interest-rate futures, swaps.
- Stocks and Bonds, trading strategies, and binomial trees.
- Derivative Securities and Risk-Neutral Valuation.
- Pricing and Modelling, numerical analysis.

8.11 System Theory and Optimal Control

Goals

This is a basic course on linear system theory and optimal control at the graduate level. Linear system theory is important as a cornerstone of control theory and is also useful in areas such as signal processing. The main goal of the course is to familiarize the student with the concepts, tools, and techniques commonly used in linear system theory and optimal control. Thus, students will not only learn the fundamental results and constructions of the theory, but will also learn to synthesize these results on their own and to produce similar results using the tools learned. The course is taught at a high level of mathematical rigor, and students are expected to understand results as well as their derivations.

Prerequisites

Linear Algebra.

References

- T. Kailath, Linear Systems, Prentice-Hall (1980).
- W.J. Rugh, Linear System Theory, Prentice-Hall (1993).
- F.M. Callier and C.A. Desoer, Linear System Theory, Springer (1985).
- O. Hijab, Stabilization of Control Systems, Springer (1987).
- R.W. Brockett, Finite Dimensional Linear Systems, Wiley (1970).
- D.G. Luenberger, Linear and Nonlinear Programming, 2nd Ed., Addison-Wesley (1984).
- P.P. Varaiya, Notes on Optimization, Van Nostrand Reinhold (1972).

Core Topics

- State equation representation: the concept of state, state equations, existence and uniqueness of solutions, linearization of nonlinear state equations, solution of linear state equations, transition matrices.
- Stability: definition of Lyapunov and asymptotic stability, conditions for stability of linear time-varying and time-invariant systems.
- Controllability and observability: definitions and theorems for linear time-varying and time-invariant systems, use of adjoint operators in proving main theorems and for deriving minimum-energy controls.
- Realization: realizability of input-output maps from the impulse response matrix or the transfer funtions, minimal time-invariant realizations, Markov parameters.

- Canonical forms: invariant subspaces, the controllable and the unobservable subspace, Kalman canonical form.
- Feedback: effects of state and output feedback on controllability and observability, eigenvalue assignment by linear state feedback, stabilization.
- Observers: full-order observers, reduced-order observers, output feedback stabilization.
- Polynomial fraction descriptions: right and left polynomial fractions, column and row degrees, McMillan degree, minimal realization.
- Linear-quadratic optimal control: fixed and free and point, finite and infinite horizon.
- Pontryagin's maximum principle (linear dynamics, nonlinear dynamics; fixed end time, free end time; minimum time problem); dynamic programming (Hamilton-Jacobi-Bellman equation).

8.12 Estimation and Filtering

Goals

Establish an adequate theoretical basis for modern communication, control and signal processing systems and make selected applications. Introduce random sequences and processes and their classification into major types. Discuss in detail two types of major importance in this application area: second-order stationary and Markov sequences and processes. Discuss selected applications: e.g., optimal (Wiener, Kalman) filtering, queueing chains, spectral estimation.

Prerequisites

Probability, Analysis II.

References

- Gray Davisson, Random Processes, Prentice-Hall, 1986.
- Larson Shubert, Prob. Models in Engr. Sci., 1979.
- Hoel, Port Stone, Introd. to Stoch. Proc., 1972.
- Papoulis, Probability, Random Variables and Stochastic Processes, 3rd ed., McGraw-Hill, 1991.
- Picinbono, Random Signals and Systems, Prentice-Hall, 1993.
- Karlin Taylor, A First Course in Stochastic Processes, Academic Press, 1975.
- Ross, Stochastic Processes, Wiley (1983).

Core Topics

- Random variables and Random Vectors: distribution function and decomposition, conditional expectation, least mean-square estimation, orthogonality principle.
- Random Sequences and Processes: classification, modes of convergence (distribution, probability, mean-square, almost sure).
- Second Order: (wide-sense) Stationary Random sequences and processes, covariance, spectral distribution and decomposition, linear, invariant operations (filtering), linear least-mean-square estimation, normal equations, rational spectral density and autoregressive-moving average models.
- Markov Sequences and Processes: Markov chain sequences, stationary and steady-state distributions, ergodicity, Markov chain processes, Kolmogoroff and Fokker-Planck equations, Poisson processes, queueing models (M/M/1).

- Independent and Orthogonal Increment Processes: Brownian motion; Wiener and Poisson processes; spectral representation of random sequences and processes.

- Applications: linear least-mean-square (Wiener and Kalman) filtering, queueing networks.

8.13 Statistical Modelling

Goals

The professional world requires manipulation and understanding of large data sets through mathematical and statistical analysis. Often the courses offered, in traditional programs, are of very theoretical nature. Many of students attending such courses do not get to see the connection with problems in industry and finance.

Prerequisites

Analysis II, Linear Algebra.

Core Topics

- Properties of Estimators: sufficient statistics, bias, consistency, efficiency; Cramer-Rao bounds, asymptotic efficiency and normality, minimum variance unbiased estimators.
- Linear Least-squares Estimation: projection theorem, properties of linear estimators.
- Hypothesis testing: likelihood ratio, Bayes' criterion, minimax criterion, Neyman-Pearson criterion, sufficient statistics, performance evaluation – receiver operating characteristics.
- Multiple hypothesis testing.
- Composite hypothesis testing: generalized likelihood ratio, uniformly most powerful test.
- Sequential detection: Wald's test.

References

- Anderson, T.W. (1958) An Introduction to Multivariate Statistical Analysis. *Wiley*: New York.
- Anderson, T. W. (1971) The Statistical Analysis of Time Series. *Wiley:* New York.
- Bellman, R. and Roth, R. (1969) Curve fitting by segmented straight lines. *J. Am. Stat. Assoc.,* **64**, 1079-1084.
- Freguson, T. S. (1967) Mathematical Statistics: A Decision Theoretic Approach. *Academic Press*: New York.
- Lehmann, E. L. (1959) Testing Statistical Hypothesis. *Wiley*: New York.
- Malinvaud, E. (1970) Statistical Methods of Econometrics (translated by A. Silvey). Amsterdam.

- Rao, C. R. (1973) Linear Statistical Inference and its Applications, 2nd ed. *Wiley*: New York.
- Savage, L. J. (1954) The Foundations Of Statistics. *Wiley*: New York.
- Searle, S. R. (1971). Linear Models. *Wiley:* New York.
- Seber, G. A. F. (1966). The Linear Hypothesis: a General Theory. Griffin's Statistical Monographs No.19. Griffin: London.
- Wald, A. (1947) Sequential Analysis. *Wiley*: New York.
- Wilks, S. S. (1962) Mathematical Statistics. *Wiley*: New York.

8.14 Stochastic Calculus

Goals

The study of random processes generated by stochastic differential equations. This is at the basis of finance, physics and electrical engineering.

Prerequisites

Analysis II, Probability.

References

- Hoel, Port, Stone, Introduction to Stochastic Processes.
- Oksendal, Stochastic Differential Equations, third edition.
- Wong, Stochastic Processes in Engineering.

Core Topics

- Stochastic Linear Systems.
- Ito calculus.
- Nonlinear SDE's and Markov Diffusions.
- Applications from finance.
- Optimal stopping and portfolio allocation.

8.15 Symbolic Computing

Goals

Symbolic computation offers tools to, e.g., manipulate polynomials: factorization, gcd computations etc, differentiate expressions and simple functions defined through programs, integrate functions and solve differential equations. Using such tools in a programmable environment leads to some of the most powerful methods brought to us by the microcomputer revolution and offers a formidable companion to numerical computation. This course will cover methods and algorithms used in doing symbolic mathematics and discuss their applications.

Both theoretical and practical aspects will be emphasized. On the practical side, sutdents will learn to be fluent in Maple (other systems will be briefly discussed), and will know how to use it both on-line, and as a software- development medium. The theoretical part will cover the basic symbolic algorithms for multiplying, expanding, factoring, and solving algebraic equations, as well as symbolic integration and symbolic summation.

Symbolic Computation is also known as computer algebra. An emerging field with many interesting applications to mathematics and science, very soon no mathematician would be able to survive without a solid mastery of its components.

Prerequisites

Basic linear algebra and calculus along with some acquaintance with computing.

References

- Computer Algebra, Davenport et. al. , Academic Press
- A Maple Primer, Frank Gravan, CRC Press
- The Maple Programming guide, by Maple Waterloo, Springer-Verlag
- A=B, Marko Petkovsek, Herbert Wilf and Doron Zeilberger, A. K. Peters (1997).
- André Heck: Introduction to Maple 2nd Edition, Springer (1996).

Core Topics

- Groebner bases
- Algebraic Identities
- Factorization schemes
- Representation
- Decomposition

8.16 Computing for Mathematicians

Goals

This is an integrative course in major software tools of current interest. We will learn the basics of UNIX, including Shell programming, HTML, website design, perl, and Java, all from a mathematical point of view.

Prerequisites

Some basic experience with internet-age computer utilization and elementary mathematics courses. The course will strive for maximal self-containment.

References

- The Unix system, Kernighan and Pike
- Java in a Nutshell, O'Reilly
- The HTML guide (downloadable version)

Core Topics

- Operating system basics: UNIX
- Shell Programming
- HTML, Java, Web Document Design
- Mathematical Interactions with Software

8.17 Cryptography and Number Theory

Goals

In the internet age data security is of paramount importance and encryption is the first line of defense. This course lays the mathematical foundations of cryptography and its links with number theory.

Prerequisites

Basic linear algebra, calculus.

References

- Cryptography: Theory and Practice, Douglas R.Stinson, CRC Press (1995).

Core Topics

- Basic number theory
- Distribution of primes
- Numerical encryption
- coding