

Performance Study of LQG, MCV, and Risk-Sensitive Control Methods for Satellite Structure Control

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Abstract

This paper will review the full-state-feedback LQG, MCV, and RS control for infinite time horizon case. In deriving the solutions of LQG, MCV, and RS control, Hamilton-Jacobi-Bellman equations are obtained using dynamic programming method. Unlike LQG and RS controllers, a pair of coupled algebraic Riccati-type equations arises in the solutions of MCV control. Average behavior of optimally controlled system is one possible performance indicator. The steady-state covariance matrices of the state and the control action are determined for finite and infinite time horizon. Furthermore, the equation for average values of the cost function is derived. A simple, single input, single output, one state example is discussed. The LQG, MCV, and RS controllers for this simple example are determined. Performance and stability characteristics of the three controllers are investigated. The performance of LQG, MCV, and RS controllers are investigated using a satellite structure control application. The objective of satellite structure control is to control the orientation of a satellite precisely and quickly. Results show that we can improve on LQG performance with both MCV and RS control.

1 Introduction

Precisely controlling a satellite structure in space is a nontrivial task. Traditionally, satellite attitude control is performed using proportional-integral-derivative (PID) controller. However, a PID controller has limited performance in certain satellite attitude control applications. There have been research activities by various researchers in regards to using different control methods such as linear-quadratic-regulator controller, game theoretic controller, and H_∞ controller for satellite attitude control [1, 8]. Figure 1 shows relationship between the cost cumulant approach and the game theoretic approach of the various optimal control methods. This paper continues to investigate and compare the use of stochastic control methods namely, linear-quadratic-Gaussian (LQG), minimal cost variance (MCV), and risk-sensitive (RS) controls, for satellite structure control.

This paper will investigate the cost cumulant approach where LQG, MCV, and RS controls are special cases. In the LQG case, the first cumulant (mean) of the cost function is minimized. In the MCV case, the second cumulant (variance) is minimized while the first cumulant is kept at a prespecified level. In the RS case, a linear combination of all the cumulants is minimized.

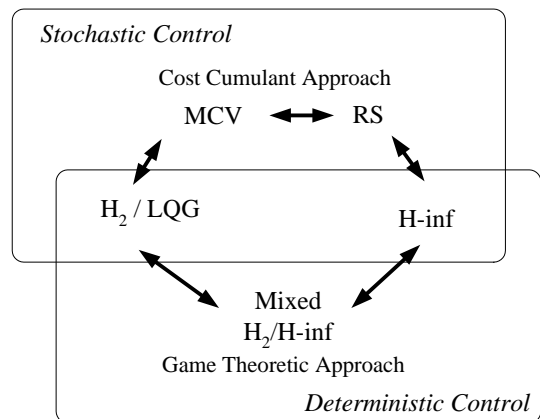


Figure 1: Deterministic and Stochastic Control Methods

2 Full-State-Feedback Infinite Time Horizon Optimal Control

We shall consider structures whose dynamic motion can be described in state variable form by the following stochastic equation.

$$dx(t) = [Ax(t) + Bu(t)] dt + E dw(t), \quad x(0) = x_0, \quad (1)$$

Here $x(t)$ is the state vector, $u(t)$ is the control vector, and $w(t)$ is a vector of Brownian motions, and $dw(t)$ is a Gaussian random process with zero mean, covariance matrix $W dt$, and independent increments.

We define a quadratic cost function,

$$\hat{J}(t_F) = \int_0^{t_F} [x'(t)Qx(t) + u'(t)Ru(t)] dt, \quad (2)$$

in which superscript $(\cdot)'$ denotes vector transposition. As is customary in such costs, the matrices Q and R

are both symmetric, with Q being positive semidefinite and R being positive definite. With these definitions we now can formulate three optimal control costs.

$$J_{LQG} = \lim_{t_F \rightarrow \infty} \frac{1}{t_F} E \left\{ \hat{J}(t_F) \right\} \quad (3)$$

$$J_{MCV} = \lim_{t_F \rightarrow \infty} \frac{1}{t_F} \left[E \hat{J}^2(t_F) - E^2 \hat{J}(t_F) \right] \quad (4)$$

$$J_{RS}(\theta) = \lim_{t_F \rightarrow \infty} \frac{-1}{t_F \theta} \log \left(E \exp \left(-\theta \hat{J}(t_F) \right) \right) \quad (5)$$

In equation (5), $\theta > 0$ corresponds to risk seeking case, $\theta < 0$ risk aversive case, and $\theta = 0$ risk neutral case or LQG case. In the sequel, we formulate and solve LQG, MCV, and RS optimal control problems using the above three cost functions.

2.1 LQG Control

Assume $R > 0$, (A, B) is stabilizable, and (\sqrt{Q}, A) is detectable. Then the optimal LQG controller for the infinite-time horizon is given by

$$u(t) = -R^{-1} B' \bar{\mathcal{P}} x(t) = -K_{lqq} x(t), \quad (6)$$

where $\bar{\mathcal{P}}$ is the solution of the algebraic Riccati equation:

$$A' \bar{\mathcal{P}} + \bar{\mathcal{P}} A + Q - \bar{\mathcal{P}} B R^{-1} B' \bar{\mathcal{P}} = 0. \quad (7)$$

Wonham proved in [10] that the algebraic Riccati Equation (7) has a unique solution in the class of symmetric, positive semidefinite maps and the map $A - B R^{-1} B' \bar{\mathcal{P}}$ is stable.

2.2 MCV Control

The problem formulation and results of finite time horizon full-state-feedback MCV control are given in [7]. Here we would like to formulate infinite time horizon full-state-feedback MCV problem as in [9]. Define

$$\bar{V}_1(x; k) = \lim_{t_F \rightarrow \infty} \frac{1}{t_F} E \{ J(t, x(t), k) \} \quad (8)$$

A function $\bar{M} : \mathbb{R}^n \rightarrow \mathbb{R}^+$, which is $C^2(\mathbb{R}^n)$, is an *admissible mean cost function* if there exists an admissible ^{∞} control law k such that

$$\bar{V}_1(x; k) = \bar{M}(x) \quad (9)$$

for $x \in \mathbb{R}^n$.

Then we minimize the cost variance of the form,

$$\begin{aligned} \bar{V}(x; k) &= \lim_{t_F \rightarrow \infty} \frac{1}{t_F} \left[E \{ J^2(t, x(t), k) \} \right. \\ &\quad \left. - E^2 \{ J(t, x(t), k) \} \right] \\ &\stackrel{\text{def}}{=} \lim_{t_F \rightarrow \infty} \frac{1}{t_F} \left[V_2(t, x; k) - V_1^2(t, x; k) \right] \end{aligned}$$

Now we admit only the bounded, Borel measurable feedback control law $k(x) : \mathbb{R}^n \rightarrow U$ such that $k(x)$ satisfies a global Lipschitz condition. We let

$$\bar{M}(x) = x' \bar{\mathcal{M}} x + \bar{m} \quad (10)$$

and

$$\bar{V}^*(x) = x' \bar{\mathcal{V}} x + \bar{v} \quad (11)$$

because the cost-to-go of any cumulant of an integral quadratic cost function for a LQG problem is affine quadratic in the state [5]. We have the following results.

Lemma 2.1 *Given a gain matrix $K(s) = -R^{-1} B' (\mathcal{M}_F(s) + \gamma \mathcal{V}_F(s))$ such that $A + BK(s)$ is exponentially stable, then $\lim_{s \rightarrow \infty} \mathcal{M}_F(s)$ and $\lim_{s \rightarrow \infty} \mathcal{V}_F(s)$ exist where $\mathcal{M}_F(s) = \mathcal{M}(s - t_F)$, $\mathcal{V}_F(s) = \mathcal{V}(s - t_F)$,*

$$\begin{aligned} -\dot{\mathcal{M}}_F(s) &= A' \mathcal{M}_F(s) + \mathcal{M}_F(s) A + Q \\ &\quad - \mathcal{M}_F(s) B R^{-1} B' \mathcal{M}_F(s) \\ &\quad + \gamma^2 \mathcal{V}_F(s) B R^{-1} B' \mathcal{V}_F(s), \end{aligned} \quad (12)$$

with the boundary condition $\mathcal{M}_F(0) = 0$, and

$$\begin{aligned} -\dot{\mathcal{V}}_F(s) &= 4 \mathcal{M}_F(s) E W E' \mathcal{M}_F(s) + A' \mathcal{V}_F(s) \\ &\quad + \mathcal{V}_F(s) A - \mathcal{M}_F(s) B R^{-1} B' \mathcal{V}_F(s) \\ &\quad - \mathcal{V}_F(s) B R^{-1} B' \mathcal{M}_F(s) \\ &\quad - 2 \gamma \mathcal{V}_F(s) B R^{-1} B' \mathcal{V}_F(s), \end{aligned} \quad (13)$$

with the boundary condition $\mathcal{V}_F(0) = 0$.

Denote $\bar{\mathcal{V}} = \lim_{s \rightarrow \infty} \mathcal{M}_F(s)$ and $\bar{\mathcal{M}} = \lim_{s \rightarrow \infty} \mathcal{V}_F(s)$. Furthermore, define $\bar{M}(t) = t_F \mathcal{M}(t)$ and $\bar{V}(t) = t_F \mathcal{V}(t)$. Then we have the following theorem:

Theorem 2.1 *In infinite time horizon, the full-state-feedback linear MCV control in $K_{\bar{M}}$ has the form*

$$k_{\bar{V}|\bar{M}}^*(x) = -R^{-1} B' (\bar{\mathcal{M}} + \gamma \bar{\mathcal{V}}) x, \quad (14)$$

where the positive semidefinite matrices $\bar{\mathcal{M}}$ and $\bar{\mathcal{V}}$ are solutions of the coupled algebraic Riccati equations:

$$0 = A' \bar{\mathcal{M}} + \bar{\mathcal{M}} A + Q - \bar{\mathcal{M}} B R^{-1} B' \bar{\mathcal{M}} + \gamma^2 \bar{\mathcal{V}} B R^{-1} B' \bar{\mathcal{V}} \quad (15)$$

and

$$\begin{aligned} 0 &= 4 \bar{\mathcal{M}} E W E' \bar{\mathcal{M}} + A' \bar{\mathcal{V}} + \bar{\mathcal{V}} A - \bar{\mathcal{M}} B R^{-1} B' \bar{\mathcal{V}} \\ &\quad - \bar{\mathcal{V}} B R^{-1} B' \bar{\mathcal{M}} - 2 \gamma \bar{\mathcal{V}} B R^{-1} B' \bar{\mathcal{V}}. \end{aligned} \quad (16)$$

Remark: The existence of the above coupled algebraic equations can be shown.

2.3 RS Control

Here we derive the infinite horizon full-state-feedback risk-sensitive control solutions for both risk-averse and risk-seeking cases. An infinite-horizon cost is defined as (5) where we shall refer to θ as the *risk-sensitivity parameter*, and $\hat{J}(t_F)$ is defined in (2). With appropriate assumptions as in [6], we find the solution of RS problem. The RS problem becomes a minimization of the cost $J_{RS}(\theta)$ over a set K_x of admissible state-feedback controls, which can be summarized in the manner

$$J_{RS}^*(\theta) = \min_{k_x \in K_x} J_{RS}(\theta).$$

To find the solution of this problem, we first find the solution to the exponential-of-integral (EOI) problem where we seek a k which would minimize the expectation of the cost function

$$\hat{\Phi}(t, x(t); k) = \exp(-\theta J(t, x(t), k))$$

where

$$J(t, x(t), k) = \int_t^{t_F} L(x(\tau), k(\tau, x(\tau)))d\tau + \psi(x(t_F)).$$

Let $\Phi(t, x; k) = E_{t,x}\{\hat{\Phi}(t, x(t); k)\}$, where the subscripts on the expectation indicate that $x(t) = x$ is given. The minimal control law k^* satisfies

$$\Phi(t, x; k^*) = \Phi^*(t, x) \leq \Phi(t, x; k).$$

Suppressing the arguments, it is known that we get the following HJB equation

$$\begin{aligned} 0 = & \min_k \left\{ \Phi_t^*(t, x) + \Phi_x^*(t, x)f(x, k(t, x)) \right. \\ & + \frac{1}{2} \text{tr} (\Phi_{xx}^*(t, x)EW E') \\ & \left. - \theta L(x, k(t, x))\Phi^*(t, x) \right\} \end{aligned} \quad (17)$$

with the boundary condition $\Phi^*(t_F, x) = \exp(-\theta\psi(x))$, where Φ_t^* and Φ_x^* denote partial derivatives.

The Hamilton-Jacobi-Bellman (HJB) equation for EOI control is given in (17). Using Fleming's logarithmic transformation method, see [3], on (17), we find the controller,

$$k(t, x) = -R^{-1}B'\hat{\mathcal{F}}(t)x, \quad (18)$$

and the Riccati equation

$$\begin{aligned} \dot{\hat{\mathcal{F}}}(t) + \hat{\mathcal{F}}(t)A + A'\hat{\mathcal{F}}(t) \\ - \hat{\mathcal{F}}(t)(BR^{-1}B' + 2\theta EWE')\hat{\mathcal{F}}(t) + Q = 0 \end{aligned} \quad (19)$$

with the boundary condition $\hat{\mathcal{F}}(t_F) = 0$ because we have assumed zero final state weighting.

Assume that there exists the limiting solution of (19), $\bar{\mathcal{F}}$, which satisfies the following algebraic Riccati equation,

$$0 = \bar{\mathcal{F}}A + A'\bar{\mathcal{F}} - \bar{\mathcal{F}}(BR^{-1}B' + 2\theta EWE')\bar{\mathcal{F}} + Q. \quad (20)$$

Here $\theta > 0$ is risk-seeking case, $\theta = 0$ is risk-neutral case, and $\theta < 0$ is risk-averse case.

Equation (20) has been studied in [6]. There he states that, for sufficiently small θ , (20) has a positive definite solution $\bar{\mathcal{F}}$ such that $A - BR^{-1}B'\bar{\mathcal{F}}$ is a Hurwitz matrix.

3 Average Performances

Assume the following:

$$\begin{aligned} E\{w\} &= 0, & E\{dw(t)dw'(t)\} &= Wdt, \\ E\{x(0)\} &= 0, & E\{x(0)x'(0)\} &= \Sigma_0. \end{aligned} \quad (21)$$

In this section we find the steady state covariance matrices for the MCV and RS control problems. Both covariance of the state and the control action are determined. Also the average value of the cost function for the finite time horizon and infinite time horizon cases are discussed. Average LQG control performance is well known, see [2, page 169] for example.

3.1 Average MCV Control Performance

Assume (A, B) is stabilizable and (\sqrt{Q}, A) is detectable. For the system described by (1), we minimize the cost variance of the cost function, $\hat{J}(t_F)$ while keeping the mean, $E\{\hat{J}(t_F)\}$, at a specified level. Then the optimal MCV controller is found to be

$$u(t) = -R^{-1}B'(\mathcal{M}(t) + \gamma\mathcal{V}(t))x(t) = -K_{mcv}(t)x(t), \quad (22)$$

where \mathcal{M} and \mathcal{V} are the solutions of the coupled Riccati equations given in Equations (12) and (13).

Theorem 3.1 *For a full-state-feedback MCV control with above assumptions, the covariance of the state, $E\{x(t)x'(t)\} = \Sigma(t)$ is given by*

$$\dot{\Sigma}(t) = (A - BK_{mcv}(t))\Sigma(t) + \Sigma(t)(A - BK_{mcv}(t))' + EWE', \quad (23)$$

with $\Sigma(0) = \Sigma_0$; the covariance of the control force is obtained from

$$\begin{aligned} E\{u(t)u'(t)\} &= R^{-1}B'(\mathcal{M}(t) + \gamma\mathcal{V}(t))\Sigma(t) \\ & \quad (\mathcal{M}(t) + \gamma\mathcal{V}(t))BR^{-1}; \end{aligned} \quad (24)$$

and average cost per unit time is given by

$$\begin{aligned} J_{av}^{mcv} &= \text{tr} \left[(\mathcal{M}(0) + \gamma\mathcal{V}(0))\Sigma_0 \right. \\ & \quad + \int_0^{t_F} ((\mathcal{M}(t) + \gamma\mathcal{V}(t))EWE' \\ & \quad \left. - 4\gamma\mathcal{M}(t)EWE'\mathcal{M}(t)\Sigma(t))dt \right]. \end{aligned} \quad (25)$$

For the infinite time horizon case, we have the following results.

Corollary 3.1 *Assume that $(A - BK_{mcv})$ is asymptotically stable. For a full-state-feedback MCV control with above assumptions, the covariance of the state, $\bar{\Sigma}$ is given by*

$$0 = (A - BK_{mcv})\bar{\Sigma} + \bar{\Sigma}(A - BK_{mcv})' + EWE'; \quad (26)$$

the covariance of the control force is obtained from

$$E\{u(t)u'(t)\} = R^{-1}B'(\bar{M} + \gamma\bar{V})\bar{\Sigma}(\bar{M} + \gamma\bar{V})BR^{-1}; \quad (27)$$

and average cost per unit time is given by

$$\bar{J}_{av}^{mcv} = \text{tr} \left[(\bar{M} + \gamma\bar{V})EWE' - 4\gamma\bar{M}EWE'\bar{M}\bar{\Sigma} \right]. \quad (28)$$

3.2 Average RS Control Performance

For now, consider the finite horizon RS control. In LQG control, we are minimizing the average value of the cost function, $\hat{J}(t_F)$, which is closely related to the minimization of the covariance of the state, x . In RS control, the average value of the cost function, $E\{\hat{J}(t_F)\}$, and the covariance of the state can be varied by choosing the appropriate risk-sensitivity parameter, θ . Consider the system described in (1). The optimal controller,

$$u(t) = -R^{-1}B'\hat{\mathcal{F}}(t)x(t) = -K_{rs}x(t),$$

minimizes the cost function (5).

Theorem 3.2 *For a full state feedback RS control with the above assumptions, the covariance of the state, $E\{x(t)x'(t)\} = \Sigma(t)$, is given by*

$$\begin{aligned} \dot{\Sigma}(t) &= (A - BR^{-1}B'\hat{\mathcal{F}}(t))\Sigma(t) \\ &\quad + \Sigma(t)(A - BR^{-1}B'\hat{\mathcal{F}}(t))' + EWE' \end{aligned} \quad (29)$$

with $\Sigma(0) = \Sigma_0$; the covariance of the control force is obtained from

$$E\{u(t)u'(t)\} = R^{-1}B'\hat{\mathcal{F}}(t)\Sigma(t)\hat{\mathcal{F}}(t)BR^{-1}; \quad (30)$$

and the average cost per unit time is given by

$$\begin{aligned} J_{av}^{rs} &= \text{tr} \left[\hat{\mathcal{F}}(0)\Sigma(0) - \hat{\mathcal{F}}(t_F)\Sigma(t_F) \right. \\ &\quad \left. + \int_0^{t_F} \left(\hat{\mathcal{F}}(t)EWE' + 2\theta\hat{\mathcal{F}}(t)EWE'\hat{\mathcal{F}}(t)\Sigma(t) \right) dt \right]. \end{aligned}$$

Thus, we note that by varying the risk-sensitivity parameter, θ , we can change the average performance. And as θ approaches 0 we get the classical LQG average performance function.

For the infinite time horizon case, we have the following results.

Corollary 3.2 *Assume that $(A - BK_{rs})$ is asymptotically stable. For an infinite time horizon full state feedback RS control with the above assumptions, the covariance of the state, $\bar{\Sigma}$, is given by*

$$0 = (A - BK_{rs})\bar{\Sigma} + \bar{\Sigma}(A - BK_{rs})' + EWE'; \quad (31)$$

the covariance of the control force is obtained from

$$E\{u(t)u'(t)\} = R^{-1}B'\bar{\mathcal{F}}\bar{\Sigma}\bar{\mathcal{F}}BR^{-1}; \quad (32)$$

and the average cost per unit time is given by

$$\bar{J}_{av}^{rs} = \text{tr} \left[\bar{\mathcal{F}}EWE' + 2\theta\bar{\mathcal{F}}EWE'\bar{\mathcal{F}}\bar{\Sigma} \right].$$

4 A Simple Example

Consider a simple system given by

$$dx(t) = (-x(t) + k(t))dt + dw(t)$$

with the cost given as Equation (3) and the covariance matrix $E\{dwdw'\} = 1dt$. For LQG case, we use the infinite time horizon version cost function (3). Solving for the positive real \bar{P} in the algebraic riccati equation (7), we obtain $\bar{P} = -1 + \sqrt{2}$ and the controller $k(t) = (1 - \sqrt{2})x(t)$. Using this \bar{P} , we obtain the average LQG performance. The average state value is 0.5946, average control action is 0.2463, and the maximum real part of the eigenvalue is -1.414 .

For MCV case, we use Equation (4). Solving the coupled algebraic riccati equations (15) and (16), we obtain

$$0 = 12 - 8\sqrt{2 + \gamma^2\bar{V}^2} + (4\gamma^2 - 2\gamma)\bar{V}^2 - 2(\sqrt{2 + \gamma^2\bar{V}^2})\bar{V} \quad (33)$$

and

$$\bar{M} = -1 + \sqrt{2 + \gamma^2\bar{V}^2}. \quad (34)$$

Thus, we choose γ and find all real positive \bar{V} using Equation (33), then we find corresponding \bar{M} using Equation (34). Then apply Corollary 3.1 to find the average MCV performance. The results are shown in Figure 2. For $\gamma = 0$, we obtain the LQG results. If we choose $\gamma = 1.1$, we have two possible solutions depending on the choice of \bar{V} . Choosing the appropriate \bar{V} , we can obtain the average state value of 0.1098, average control action of 4.442, and the maximum real part of the eigenvalue of -41.45 , which gives 81.5% improvement of the average state value over the LQG case. Note that it is possible to have multiple solutions for given γ . Also $\gamma = 0$ corresponds to the LQG case, thus improvements in average performance is possible. For RS case, we use the cost function (5). Once again we solve the algebraic Riccati equation (20), and obtain $\bar{\mathcal{F}} = \frac{1 \pm \sqrt{2 + 2\theta}}{-(1 + 2\theta)}$. Now, we find

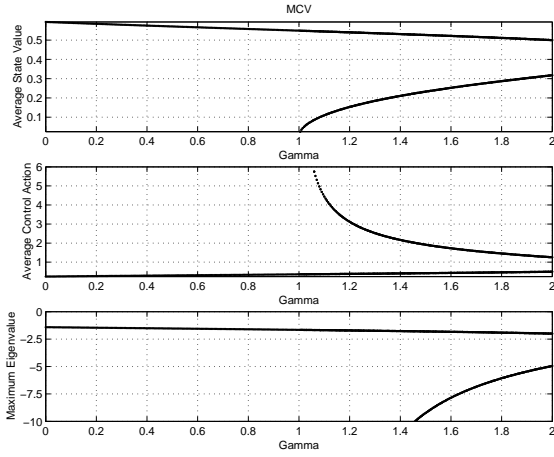


Figure 2: Average MCV Control Performance

the range of θ that will give positive $\bar{\mathcal{F}}$. We determine that for $-1 < \theta < 0.5$, there are two real positive values given by $\bar{\mathcal{F}} = \frac{1 \pm \sqrt{2 + 2\theta}}{-(1 + 2\theta)}$. For $\theta > -0.5$ only $\bar{\mathcal{F}} = \frac{1 - \sqrt{2 + 2\theta}}{-(1 + 2\theta)}$ gives a positive real solution. Thus, valid controller is given as follows.

$$\text{Controller 1: } k(t) = \frac{1 - \sqrt{2 + 2\theta}}{-(1 + 2\theta)} x(t)$$

for $-1 < \theta$ except for $\theta = -0.5$ and

$$\text{Controller 2: } k(t) = \frac{1 + \sqrt{2 + 2\theta}}{-(1 + 2\theta)} x(t)$$

for $-1 < \theta < -0.5$. Figure 3 and 4 show the average performance of the RS Controller 1 and 2, respectively. In Figure 3, $\theta = 0$ corresponds to the LQG case. Thus we note that for $\theta < 0$ we can obtain smaller average state value than LQG. Figure 4 shows more dramatic improvements over LQG. For example for $\theta = -0.55$, the average state value is 0.1562, average control action is 3.044, and the maximum real part of the eigenvalue is -20.49 . Thus we can achieve 73.7% improvement over the LQG case in the average state value. The price that we have to pay is in control action required to achieve that performance. The figures also show that stability margin also improves with appropriate choice of θ .

5 Satellite Structure Control Application

A remote sensing satellite, KOMPSAT-I's nonlinear and linear model is developed in [8]. The linear model is given as $dx = (Ax + Bu)dt + Edw$ where A, B , and E are given in [8]. Using this linear model and parameters of KOMPSAT-I, we compare performances of infinite horizon LQG, MCV, and RS control. Here the state vector, x consists of roll, pitch, yaw Euler angles;

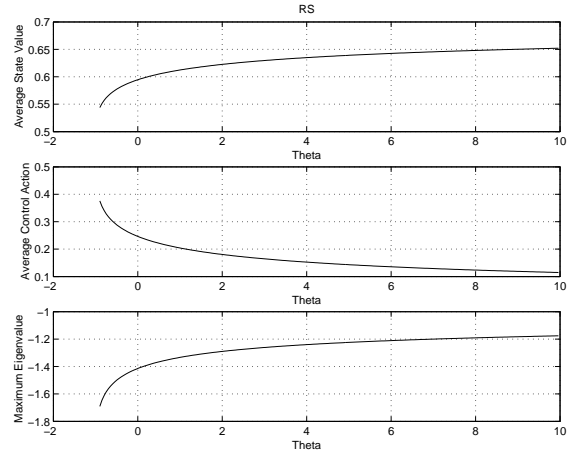


Figure 3: Average RS Control Performance Using Controller 1

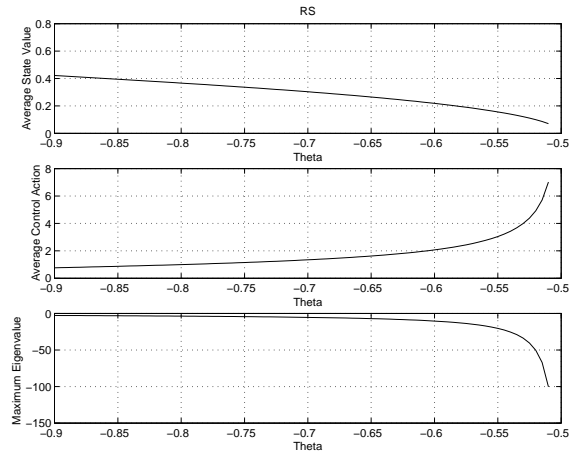


Figure 4: Average RS Control Performance Using Controller 2

x, y, z , components of the angular velocity vector; and reaction wheel 1, 2, 3, 4 speed vectors. The control vector, u is absolute torque due to the reaction wheels and torque due to the thrusters. The disturbance vector, w is the torque due to the aerodynamic atmospheric drag, torque due to the magnetic field, and torque due to the solar radiation pressure, which are assumed to be Gaussian. We used pole placement technique to find initial K for MCV control.

Figure 5 shows the average roll angle of the satellite as the parameters γ and θ varies. Note that both controller performs better than classical LQG controller, where $\gamma = \theta = 0$ corresponds to the LQG case in all the Figures. Figure 6 shows the stability margin variation with respect to the two parameters. These graphs show that RS and MCV controller are very comparable. To compare these two controllers, we matched the Euler roll angle of the two controllers and compared the rest of the values. The results are given in Table 1. Both RS and MCV perform better than LQG in average Eu-

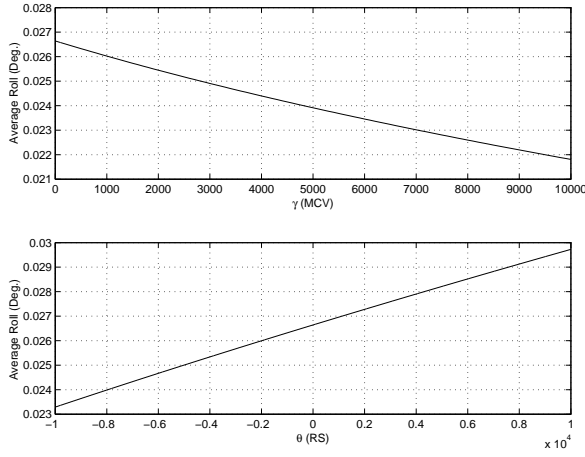


Figure 5: Comparison of Roll Euler Angle Using LQG, MCV, and RS Control

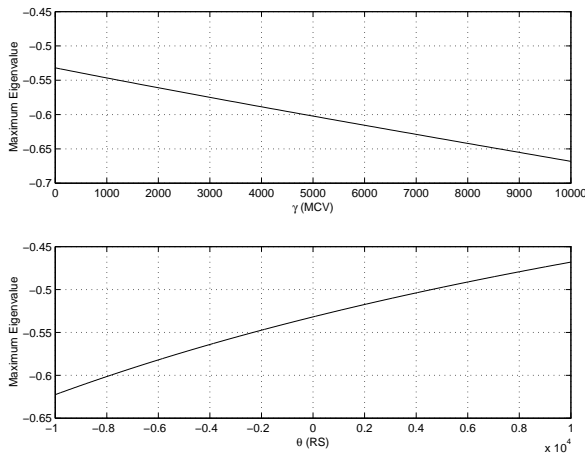


Figure 6: Comparison of Stability Margin

ler angles and on the stability margin. The table shows that RS and MCV performance and stability are very close when average roll angle is matched.

	RS ($\theta = -7940$)	MCV ($\gamma = 4800$)
Roll	$2.401 \cdot 10^{-2}$	$2.401 \cdot 10^{-2}$
Pitch	$2.804 \cdot 10^{-2}$	$2.803 \cdot 10^{-2}$
Yaw	$2.570 \cdot 10^{-2}$	$2.569 \cdot 10^{-2}$
RW1	$4.704 \cdot 10^{-4}$	$4.704 \cdot 10^{-4}$
Thruster x	$8.998 \cdot 10^{-2}$	$8.995 \cdot 10^{-2}$
Thruster y	$1.134 \cdot 10^{-1}$	$1.134 \cdot 10^{-1}$
Thruster z	$9.879 \cdot 10^{-2}$	$9.877 \cdot 10^{-2}$
Max. Eigenvalue	$-6.009 \cdot 10^{-1}$	$-5.995 \cdot 10^{-1}$

Table 1: Comparing RS and MCV when Average Roll Angles are Matched.

6 Conclusions

We compared performance and stability of LQG, MCV, and RS controllers. Both MCV and RS controller has a parameter that can vary to obtain better performance and stability margin. Thus MCV and RS controllers can outperform an LQG controller simply by varying this parameter. MCV and RS controllers showed comparable performance and stability margins.

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